1. **Answer: B. Additional Comma Uses and Misuses**
The original text is incorrect because it includes a comma in a place used to separate articles in a list. Lists must be comprised of 2 or more articles. For example, were it “a snake curving, winding, and bending,” it would be necessary. D similarly baits this mistake, however “snake” is not an article in a list. Imagine a narrator reading this dramatically. Where would he pause?

2. **Answer: J. Joining and Separating Sentences: Comma + FANBOYS**
Note the “NOT” in this question. J is the only option which does not offer any form of conjunction for the double predicate in this sentence. “Cleared this path,” and “Paved it with packed gravel,” are both predicates, independent of one another, which share a subject. In order to compound them, they must be separated with a comma, conjunction, or similar, as provided in options F-H.

3. **Answer: D. Pronouns: Agreement and Case**
The most tempting answer here is C, however it is incorrect because adding a “that” does not actually fix the problem with the word “they” in this sentence. In this context, the antecedent to the pronoun “they” would be “the county,” which would mean the governing body itself was peacefully hiking and biking. J offers a more general answer, however generality trumps specificity when the specifics are wrong.

4. **Answer: F. Dashes**
Em dashes are tricky, but don’t be intimidated. If you aren’t sure whether it’s being used correctly, instead look at the options and see which you know are wrong. G can be ruled out because a semicolon can only lead into an independent clause, however here we have a fragment. H should be a fairly clear run-on sentence with no punctuation whatsoever. J is wrong for the same reason as G yet with a superfluous comma thrown in. Even if you don’t quite understand semicolons you can solve this question.

5. **Answer: B. Is it Relevant: Deleting**
It may be a good idea to read ahead a little bit here; it’s hard to say what the essay could lose if you don’t know what it has to begin with. Then consider the sentence in question. It doesn’t provide a reason for the narrator being in the forest, so A is out. It certainly doesn’t contrast the lightheartedness; in fact it reinforces it. D is one that can’t be ruled out because it doesn’t make a direct claim, however when compared to B it’s fairly clear which makes sense—otherwise he may as well be riding a green plumber.

6. **Answer: J. Joining and Separating Sentences: Comma + FANBOYS**
Here it is important to include a conjunction for the compound sentence. Options G & H both have a transition; however, they would only be correct at the start of a new sentence. If there was a period in the place of the comma they may be feasible, however only J (which does not refer to the previous sentence by means of a pronoun) is concise and without redundancy.

7. **Answer: D. Word Pairs and Comparisons**
This question can be frustrating because it’s not an objective one, but rather depends on your ability to read the connotation of the words used. The text as written is entirely neutral in tone, so we can be sure it begs improvement. B and C are both somewhere on a scale from neutral to fond, however D matches the affectionate tone sought in the question.
8. **Answer: F. Pronouns: Agreement and Case**

G & H are grammatically correct options, however in the context of the passage they do not make sense. “You” and “one” do not match the first-person narration, so can be eliminated. Unlike the former two, J is not grammatically correct because the addition of “while” would suggest a new independent clause to be added at the end.

9. **Answer: A. Joining and Separating Sentences: Comma + FANBOYS**

This is a compound sentence, with two independent clauses joined by a comma and the conjunction “and.” B & D both cause the first clause to become dependent, thus breaking the sentence. While C is grammatically correct, in the context of the sentence it is far less reasonable than the original text.

10. **Answer: H. Joining and Separating Sentences: Comma + FANBOYS & 10, Adjectives and Adverbs**

“I bounce gently along in my padded chair” is an independent clause, a complete thought, so it needs no comma whatsoever. If you read the sentence to yourself, you will likely notice that there’s no natural pause in the underlined portion. If the commas confuse you, you can still note that only options H & F correctly use an adverb instead of an adjective.

11. **Answer: A. Is it Relevant: Replacing**

In order to answer this question, you have to first find the subject of the next paragraph. Noting the emphasis on the narrator’s leisurely pace, it makes sense to begin with a reference to speed. The word “their” a bit further on also provides a hint in that it is a plural pronoun; “Nature” and “The sun” are singular, so they cannot be correct answers. Given the choice between cyclists and days, it makes far more contextual sense to pick the former.

12. **Answer: H. Joining and Separating Sentences: Period**

Here we must determine whether the portion after the underline is an independent clause or a dependent clause. To do this, all you need do is read it on its own. “I don’t understand their hurry.” is a complete sentence, so it is an independent clause. As such, the answer is to make it a new sentence. If there were an option for a semicolon that could also be correct, however the other options are not.

13. **Answer: D. Parallel Structure: Lists & 5, Additional Comma Uses and Misuses**

Here, again, we must find the potential clauses. While “I spend an hour,” is an independent clause, “looking and listening and learning,” is not. It cannot come after a semicolon, then, so we know right away there must be a change. C is no better, so we must look to B & D. The commas in option B are superfluous and incorrect, so foregoing punctuation to create one long clause is the answer.

14. **Answer: G. Apostrophes: Possessive vs Plural**

Singular possessive. There is one trail which has one end. As written it is a plural without possession, H is a plural possessive, and J isn’t even a thing. G is the correct singular possessive.

15. **Answer: B. Suppose the Writer’s Goal**

The trick with this question is that the goal described in the question is not actually a great fit for the essay. However, when we look at the “No” options it’s clear that neither is appropriate. C wrongly asserts a focus on the city, which is blatantly irrelevant, while D suggests a technical focus on the wheelchair, Luigi. Both of these are flawed reasoning, so we must discern which “Yes” is more appropriate. A also offers an invalid explanation, while B gives a general yet correct evidence. If you read the passage, it should be easy to determine which answers are not relevant to the text.
16. Answer: G. Non-Essential Transitions, Words, and Clauses
If we recognize the portion of the sentence leading up to the semicolon is not an independent clause, we can rule out both answers F & H. The difference between G & J is a subtle one, given the heavily-punctuated sentence: the final comma. It does not make sense to have a comma after “thousands,” as it is part of the subject “thousands of New York City residents,” so it can be determined invalid. G is the remaining answer.

17. Answer: C. Colons
The colon is an interesting creature. It is used to present a list, example, or other subject. In this case, we are being introduced to “an incredible engineering feat.” A semicolon is not correct because we do not have an independent clause, nor can we forego punctuation as in D because we are clearly not in the same idea. The preposition “over” in option B would give the sentence a new meaning, suggesting that the city was celebrating literally above the completed subway.

18. Answer: G. Is it Relevant: Deleting
This question relies on your comprehension skills. When we consider what explicitly can be determined from the phrase in question, only G is a reasonable answer. It could be argued that one might infer the other answers, however the test won’t listen to your arguments: only one answer is definitely correct.

19. Answer: D. Shorter is Better: Redundancy and Wordiness
When deletion is an answer, always give it special attention; it is often correct. In this case it is, however we still must determine why. The beginning of the sentence already states that the route took 26 minutes, so adding that it was under a half an hour is redundant. The test hates redundancy, so you can be sure it is better off omitted.

20. Answer: F. Is it Relevant: Replacing
The answer here lies in the question. Which choice CLEARLY and EFFECTIVELY describes the solution? Options G-J are all extremely vague, and though not in breach of any grammar rules they are far from what the question sought.

21. Answer: B. Pronouns: Agreement and Case
In order to determine the correct pronoun, we must consider the antecedent (the implied noun to which the pronoun refers). The word “that,” as used in the text, can only be referring to “proposals.” This does not make sense in the context of the sentence. “Those” as in C has the same problem. We cannot delete as in D, because then “took” would have no subject. “It,” though vague and with no clear implied antecedent, is correct because it refers to the process of resolution mentioned later in the sentence. This is a difficult question from a technical standpoint, but if you read aloud to yourself you will likely be able to determine which just sounds right!

22. Answer: J. Non-Essential and Essential Clauses
This sentence has a fairly simple subject and predicate. “The engineer William Barclay Parsons,” is the subject, and “accepted responsibility for overseeing this project,” is the predicate. There is one other way to punctuate this sentence, but it is not one of the options. If the em dash in G enclosed BOTH sides of the name, it would be a correct notation for a descriptive aside. Similarly, commas could be used on either side. HOWEVER as we have no option for this, we know to keep the subject as a whole, and the simple sentence is in no need of punctuation.
23. **Answer: C. Adjectives and Adverbs**
   This question tests your understanding of parts of speech. What we need here is an adjective and a noun. This would fit the preceding sentence most simply. “Innovation,” is a noun, however, so as is we have noun-noun. B offers verb-noun. D is verb-noun. Only C has an adjective describing a noun for a correct companion to the verb “using.”

24. **Answer: F. Shorter is Better: Redundancy and Wordiness**
   More complicated does not mean more correct! Options G and H are simply messes of words put there only to be confusing and make you doubt the correct, simple answer. Note that in J omission is not correct, because in this case we lose an important detail. While the sentence could end here, it is not the same as specifying what they were digging.

25. **Answer: C. Colons**
   While the start of this sentence (up to “created”) is an independent clause, the fact that we must include the word “with” in all answers does rule out many options. The text as is incorrectly uses a colon without introducing a subject, while D tempts one with an appearance of beginning a list. In reality, however, the list does not begin with the word “layers.” As it is describing all of the materials as being layers, a colon could only be used after the word “of.” B incorrectly places a comma between “created” and “with,” which are more-or-less conjoined at the hip. No punctuation is needed here to complete the sentence.

   Many do not understand the correct use of the word “whom.” As a general rule of thumb, it will almost always follow the word “of.” You will never see “of who,” except in very specific corner cases. “Them” is incorrect because the antecedent would be the subject of the sentence, “stations.” J may be tempting because of the potential of “many” to be a noun meaning “many people,” however a comma would not be correct if we sought to use it as such. Beginning a new independent clause is possible, but not an available answer.

27. **Answer: A. Non-Essential Transitions, Words, and Phrases**
   “Therefore” means “because of what was just said.” “For instance” is specifying something as an example. “That is” is a clarification. “However,” however, is adding that something is contrary to what was just presented. Here we are told that it many were skeptical, but then, contrary to this statement, we are told they adapted quickly. “However” is the best, most logical answer.

28. **Answer: G. Is it Relevant: Inserting**
   Which answer clearly and effectively portrays the scale in the present day? F offers no reference to scale though it does refer; to the quote H refers more to a feat of management than of magnitude, but is a closer option than the others; it does not, however, flow from the paragraph. J has a similar problem in that, while directly about size, it has nothing to do with the newspaper quote. Only G successfully integrates the quote to modern statistics to support the paragraph.

29. **Answer: C. Paragraph Order & 9, Pronouns: Agreement and Case**
   The clue here should be the pronoun “this.” Where is there a logical antecedent just before that this could follow? Only C describes a technique, so it must logically go there.

30. **Answer: J. Word Pairs and Comparisons**
   This vocab question directly asks for you to interpret the connotation of the words provided. The existing word and option G are both very neutral, and in no way suggest the previous decline. While “influenced” is a more vivid word, it also does not reference the decline addressed in the question. “Revived,” however, implies it had been dying (thus, in need of revival).
31. **Answer: B. Verbs: Agreement**
   We can infer that the “elite galleries” and the “upper class” are the same or similar group of people, so some answers do not make sense. A, C, and D all suggest he brought his art out of the upper class by or while catering to it, which hardly makes sense. Instead, the pronoun in B changes the verb “cater” to be done by the galleries instead of Rivera, making a logical sentence.

32. **Answer: J. Sentence Order**
   This is a complicated question and is very intimidating at first glance. However, if you read each answer all the way through you will find it’s much easier than it looks. G is immediately eliminated because it is in the present tense with the active verb being “should.” Now that we have confirmed the root “Rivera attracted,” we can ask: what did he attract? Controversy, of course. And why? For his belief. To expand, his belief that the working class should wield more political power. This all fits together in answer H.

33. **Answer: D. Shorter is Better: Redundancy and Wordiness**
   As always, when deletion is an option make sure you consider it strongly. Here it is correct, because it is already specified to be his “artistic subject,” so adding that it is in his art is redundant. Always cut redundancy when you can!

34. **Answer: H. Parallel Structure: Prepositions**
   “As” is a comparative word which implies equality. It’s use before the word “expansive” means we are about to be told something equally expansive. Because two equal things cannot be more or less equal than the other, “than,” which implies this inequality, is not correct. “If” mandates a condition, and there is no condition in this comparison. “Then” denotes time or sequence, which is also irrelevant. Another “as” properly notes the equality of the comparison.

35. **Answer: B. Verbs: Agreement**
   The action verb “depict” here is our clue. “Depict” must have a plural subject, while “depicts” would be the singular form. This also rules out option D. Though harder to tell at first look, C is also singular. “Each of his frescoes” would mean any one specific fresco out of a large grouping. “Many” is the only plural option, and thus the only one which accurately pairs with the verb in the sentence.

36. **Answer: H. Non-Essential Clause, “…,…”**
   The core subject and predicate here are respectively “The same workers” and “stand tall.” The rest is descriptive, and in order to determine how they fit together in the given order we have to recognize which parts are and aren’t part of the subject. “On the right. . .through history” is a description of the workers. It is not an independent clause, and thus cannot retain the period nor use a semicolon. Em dashes also follow this rule. While it is clumsy and loaded with commas, using one to separate the description from the noun is correct here.

37. **Answer: C. Verbs: Agreement and Tense**
   The plural “were” immediately rules out leaving the sentence unchanged. The addition of “if” in both B & D actually causes the predicate to become part of the subject by turning it into a descriptor, causing the sentence to become a fragment. The simple C is correct and concise.

38. **Answer: F. Additional Comma Uses and Misuses**
   Because there are only two adjectives here, not the three required to make a list, there does not need to be any punctuation. If there was one more adjective it would be necessary to punctuate, but as is the sentence is correct.
39. Answer: C. Is it Relevant: Deletion
What detail is provided in the underlined portion? Is it elsewhere in the sentence? These are the two questions to consider when given an omission question. Because it is not elsewhere, we immediately know A to be untrue. B is not true because the sentence is still complete without the knowledge of what is being painted. D is incorrect because, while one could argue it is unnecessary, it is in no way ambiguous. C correctly points out that the detail is new and relevant.

40. Answer: J. Joining and Separating Sentences: Comma + FANBOYS
Choosing the correct conjunction is entirely dependent on context. "When" is not correct because there is never a time when the South central panel shows something different. "Since" is also incorrect because the South central panel did not cause the North to show what it does. "Thus" also incorrectly implies causation. "And" correctly joins the two ideas without implying one’s influencing the other.

41. Answer: B. Verbs: Agreement
We must have a predicate for this sentence to be complete. Here, "depicting" serves not as an action verb but simply part of a description. C and D similarly fail to create action. Only B creates a predicate, and thus a complete, independent clause.

42. Answer: F. Is it Relevant: Inserting
What matters here is not whether it should or should not be added, but rather the reason provided. Even if you think it should not, H & J are not sufficient reasons. J is close, however the examples are not a true digression. G similarly offers incorrect evidence.

43. Answer: B. Non-Essential Clause, “…”
The portion between the underline and “celebrates” is a distractor. The description offered cannot be changed, and needn’t impact your answer. Instead imagine the sentence moving directly from “because” into “celebrates.” The only answer that makes sense is "The fresco is a dynamic work that celebrates all working men and women."

44. Answer: J. Non-Essential Transitions, Words, and Phrases
As is often the case, the deletion is correct here. One hint to this aside from the fact that none of the other answers make much sense in underline is that they all mean more or less the same thing; they all lead into an exception or something contrary to what was just said. This doesn’t fit here, and even if it did they couldn’t all be correct. Thus, delete.

45. Answer: A. Sentences and Fragments
You hopefully notice when first reading the sentence that there are no glaring errors; that is, it sounds correct. Often this gut feeling is right. Note that “even though we attended different high schools and colleges” is not an independent clause. It can’t be separated by a period or semicolon as in C & D. The addition of “yet” can be deemed unnecessary because “even though” already serves the purpose of connecting these ideas. “Even though” could be replaced with “yet,” however having both is not correct.

46. Answer: J. Joining and Separating Sentences
Be careful! This is a “NOT” question. If you don’t read carefully you may see that F is grammatically correct and immediately fill it in. Hopefully you would read the other answers and notice that all except J are, because J is the only one which does not in any way join the compound sentence.
47. **Answer: C. Additional Comma Uses and Misuses**
This can be a difficult comma question. When stuck on comma questions, reading aloud with long pauses for commas helps to determine what is correct. We can rule out B and D fairly quickly because they do not offer adverbs, and “enjoyed” is already the active verb. Between B and A, you can hopefully tell that two commas sound better than none. Grammatically, there does not need to be a “though,” in this sentence at all: that’s how you can tell it should be separated with nested commas.

48. **Answer: F. Word Pairs and Comparisons**
We need a phrase here that expresses “fondness” and shows a “positive reaction.” G and H are very neutral, with almost no expression. J expresses surprise, which though not exclusive to fondness is nowhere near as direct as the written sentence.

49. **Answer: B. Pronouns: Agreement and Case**
“Whom” is used with prepositions. Here we do not have one, so “who” is appropriate. “Whose” is possessive, which clearly is not right. “Which” does not refer to people, even, so a simple “Who” is the correct answer.

50. **Answer: F. Pronouns: Agreement and Case**
Here we have to determine which pronoun implies the most appropriate antecedent. “We,” would mean who, the speaker and his parents? That doesn’t make sense, seeing as they told him about her. “They” would imply his parents learned through themselves. “He” makes less sense yet, as there’s no singular male referred to here. “I,” we can assume, is correct, and is similarly consistent with the first person narrative.

51. **Answer: B. Sentence Order**
The biggest hint here should be in sentence 2, where the reference to Joan is unprompted. It would make more sense following sentence 3 explaining how he found her, which itself makes more sense once Fairbanks is introduced in sentence 1. These questions can be tricky little puzzles, but if you look out for signal words they become much simpler.

52. **Answer: J. Shorter is Better: Redundancy and Wordiness**
In this question, deletion is again correct. The most common reason for deletion is redundancy, and this sentence has it. “To keep the battery from freezing” already implies “so the battery would stay warm.”

53. **Answer: D. Word Pairs and Comparisons**
All of these are viable past-tense notations for “shine” except “shoned,” which is not a word. The hybrid of “Shined” and “Shone” is meant to seem like another possibility.

54. **Answer: H. Is it Relevant: Deleting**
The detail in question, we can see, is a descriptive one which offers a setting and some simple imagery. It does not include a response to the weather, nor does it analyze the plugging. While you may find the detail unnecessary, it does not repeat anything, and you can’t choose half of an answer. Thus, H is the most correct option.

55. **Answer: A. Word Pairs and Comparisons**
All answers here are grammatically correct, so you must determine which makes the most sense and conveys the author’s thoughts most accurately. B implies they found the habits by chance, C that they paid the habits a visit, and D that they left without the habits and came back with them. “Slipped into,” correctly suggests they unknowingly recommenced their old habits.

56. **Answer: H. Sentence Order**
As is, the sentence states Joan’s house was on top of the rabbit hutch. G says the field is on the hutch. J also places the field atop the hutch. Only H accurately places Joan and the narrator on the hutch.
57. Answer: B. Word Pairs and Comparisons
All of the options except B show Joan’s being extremely occupied by her work. B seems to say her work gained possession of her.

58. Answer: H. Parallel Structure: Prepositions
“Along” suggests positioning, which is not applicable here. The other words all show that her decisions were the subject of her firmness, which “Along” merely places her firmness somewhere therein.

59. Answer: A. Is it Relevant: Replacing
While the changes listed are correct sentences, they do not aptly sum up the passage. If you read, you’ll know there was no hint of the “sadly” in B, nor a focus on a mutual devotion to work, nor indeed any real explanation of the time spent together when they were young.

60. Answer: G. Paragraph Order & 18.1, Is it Relevant: Inserting
The best answer is G because the suggested sentence is a transition from the description of their bond to their eventual reunion. Words like “yet” and “such” are excellent indicators of where to place sentences in these questions.

61. Answer: A. Shorter is Better: Redundancy and Wordiness
The word “attaching” needs two nouns; one to be attached, and one to which to attach. Because we already have “began attaching,” we do not need the additional actions of “which they connected” and “which they related” as offered in the answers. Answer D does not offer the second noun to which to attach the stories. Thus, only A is a valid answer.

62. Answer: H. Joining and Separating Sentences: Comma + FANBOYS
This is actually two independent clauses, best written as a compound sentence. “The sun sets,” is an entire idea, and “The images of...appear in lights overhead,” is similarly so. They can be connected with a simple comma and conjunction. J could be a grammatically correct sentence, however it would mean the sun was setting the images; the sun, not holding them up, could hardly set them.

63. Answer: A. Non-Essential Clause, “...,”
In this case, “or constellation” is offering an alternate wording. As it is not a functional part of the subject-predicate relationship, it is best nested between two commas. If you are unsure when a phrase is one such extra detail, see if the sentence makes sense without it. If it does, you are likely safe to put it in some commas (or em dashes, if they’re an option).

64. Answer: H. Is it Relevant: Replacing
Read ahead! It’s okay to skip a question like this and come back to it when you have the context. Near the end, a whole paragraph is devoted to the myth and constellation of Orion, so this question becomes simple if you have a little patience.

65. Answer: D. Word Pairs and Comparisons & Is it Relevant: Replacing
Don’t be fooled by the specification “in modern times!” The question is looking for visual information, which is only provided in option D. While not explicitly referencing time periods, it is still the correct answer to the question.

66. Answer: G. Word Pairs and Comparisons
This is a question of vocab. “Overseers” are people in a managerial position who monitor the actions of others. Each other option here describes someone who looks at the sky.

67. Answer: A. Word Pairs and Comparisons
The significance comes from the detail of the Karasuki representing a plow. Plows being a traditional agricultural tool, it’s relevant that the Japanese communities mentioned are agricultural ones. Their distance, population, and historical-ness have no pertinence to the plow imagery.
68. Answer: J. Diction and Register
For a formal, informative essay, it’s not appropriate to use such casual phrases as in F and G. H, while not such a breach of etiquette, is wholly unnecessary, and it is a faux-pas to use second-person pronouns in an informative essay.

69. Answer: A. Non-Essential Transitions, Words, and Phrases
B is simply false, while C & D both leave out important details. It is significant that this interpretation is local to other parts of Japan, and not those already mentioned.

70. Answer: G. Verbs: Agreement
“Stars” is the root of the subject here, and is plural. “Has” is singular; “could of” is wrong not only because it doesn’t fit, but also because it would be “could have” if it was even apt; “has been” is just as singular as F. Only G has a plural verb and actually makes sense.

71. Answer: D. Non-Essential Transitions, Words, and Phrases
This one sounds really weird, and you may be tempted to throw commas in there just because it seems like a run-on. But don’t let your ears fool you; there is no need for a comma until the end. “The mythology of the Tswana people of South Africa” is one long subject, and “In” is necessary for the word “represent” later in the sentence. Imagine it flipped this way: “These same stars represent three pigs in the mythology of the Tswana people of South Africa.” The only comma we need is to show where we flipped.

G and H are not grammatically correct, however J and F are. To determine which is correct, we have to use logic to analyze what each is saying. F seems to say that the scorpion’s inability to sting is the cause of the distance, while J says the distance prevents the stinging. J makes more sense in this context.

73. Answer: C. Apostrophes: They’re, Their, There
Their, there, they’re: the banes of English-speakers.

74. Answer: G. Joining and Separating Sentences: Comma + FANBOYS
This is a compound sentence! As with all the others in this test, it can be joined with simply a comma and an “and.” Note that “span” is not “spanning,” which would validate other options.

75. Answer: D. Paragraph Order
The important part here is “free of direct references to a specific culture’s view of the three stars.” Dividing anywhere earlier than sentence 7 would leave some references in the conclusion, and thus be incorrect.
1. **A. Linear Function: Rate**
$20$ per $v$, vehicles; $10$ per $p$, persons

   \[ 20v + 10p \]

2. **F. Solving Equations**
   \[
   (9 + 5 - (-6))(5 + (-6)) \\
   (20)(-1)
   \]

   \[ F. -20 \]

**Tip:** pay attention to NEGATIVE signs. When substituting and distributing negative numbers ALWAYS use parentheses.

3. **E. Rate & Proportion**
   1st: $60$ per min. $\times 80$ min $= 480$
   2nd: $80$ per min. $\times 60$ min $= 480$

   Total $= 480 + 480 = 960$

   \[ E. 960 \]

4. **J. Mean, Average**
   You know how to do an average. When asked to keep the average the same, the next value added must simply be equal to the average!

   \[
   \frac{(210 + 225 + 254 + 231 + 280)}{5} = 240
   \]

   \[ J. 240 \]

5. **C. Rate & Proportion**
The ACT loves to ask about hourly wages (pay per hour). Here she makes $7.50$ per hour for 40 hours.

   $7.50 \times 40 = 300$

   And 1.5 times for more than 40 hours.

   $7.50 \times 1.5 \times 2 = 22.50$

   $300 + 22.50 = 322.50$

   \[ C. 322.50 \]

6. **K. Word Problems: Translation & Vocabulary**
   “A number squared” is $x^2$
   “is 39 more” is “$= 39 +$”
   “product of 10 and $x$” is $10x$

   \[ K. x^2 = 39 + 10x \]

7. **E. Solving Equations**
   \[
   9(x - 9) = -11 \quad \text{Distribute 9} \\
   9x - 81 = -11 \quad \text{Add 81} \\
   9x = 70 \quad \text{Divide by 9}
   \]

   \[
   x = \frac{70}{9}
   \]

   \[ E: \frac{70}{9} \]

8. **H. Rate & Proportion**
   Given: $4.00$ is discount price. Spent $60$
   $60/4 = 15$ tickets purchased
   Discount is $37.50$ less
   \[
   \frac{($37.50 + $60.00) / 15 = \$6.50}
   \]

   \[ H. \$6.50 \]

**Trap:** $F$ is only the discount per ticket.

9. **A. Factoring & FOIL**
   FOIL! (only if you have to)

   \[ A. 9x^2 - 16y^4 \]

   The ACT loves the Difference of the Squares. A skilled test-taker should be able to recognize these backwards and forwards.

   **Tip:** Memorize
   - $(a - b)(a + b) = a^2 - b^2$
   - $(a + b)^2 = a^2 + 2ab + b^2$
   - $(a - b)^2 = a^2 - 2ab + b^2$
10. J. Quadrilaterals
Area. \( l \times w = 32 \)
Perimeter \( 2l + 2w = 24 \)

Algebra solution
Solve for \( w \), by substitution.
\( 2l + 2w = 24 \)

Subtract \( 2w \) from both sides
\( 2l + 2w - 2w = 24 - 2w \)

\( 2l = 24 - 2w \)
Divide both sides by \( 2 \)
\( \frac{2l}{2} = \frac{24 - 2w}{2} \)
\( l = 12 - w \)
Substitute \( 12 - w \) for \( l \) in \( l \times w = 32 \)

\( (12 - w) \times w = 32 \)
\( 12w - w^2 = 32 \)
\( w^2 - 12w - 32 = 0 \)

Factor
\( (w - 8)(w - 4) = 0 \)
\( w = 4 \text{ or } 8 \)

J. 4

That's a heck of a lot of algebra. Guess and check is better. Because the area and the perimeter are integers, then the length and the width both have to be integers, since:
Integer \( \times \) Integer = Integer
Integer + Integer = Integer

So what are the factors of 32?
8 \( \times \) 4 \& 16 \( \times \) 2.

Guess and check.

Trap: K: 8, don’t make the mistake of putting the longer side.

11. D. Triangles
The SUM of the angles is 47°, then the other is simply \( 180° - 47° = 133° \)

D. 133°

Trap: B. The measure of A and B added is 47, if you make each one 47, then there sum is 94, making C 86.

12. K. Counting, Permutations, & Combinations
This is straight up counting. To make a lunch you need one of each, so just multiply the number of options together.
\( 3 \times 3 \times 4 \times 2 = 72 \)

K. 72

Traps: F: 2 is the average of the numbers and H: 12 is the sum. Don’t make these mistakes.

13. B. Properties of Integers
Consecutive integers can be represented as
\( (n, n + 1) \)
\( n + 3(n + 1) = 79; \) solve for \( n \).
\( 4n + 3 = 79 \)
\( n = 19 \)

B. 19, 20

Trap: Who says the ACT is not tricky? E is a trap of two consecutive numbers whose sum is 79.

14. F. Functions \( f(x) \)
This is a very basic question that is on every test: evaluate a function for a given value of \( x \).
\( f(-3) = -8(-3)^2 \)
\( f(-3) = -8(9) \)
\( f(-3) = -72 \)

F: -72

Trap: WATCH OUT FOR NEGATIVE SIGNS!
On every ACT, there are simple substitutions and evaluations that include operations with negative numbers. They love subtracting and distributing negative numbers, because you do not.
15. **C. Exponents & Roots**

\[ 3^x = 54 \]

Look at the answers MUST be true.
You have memorized the powers of 3 (because I told you to).
\[ 3^2 = 9; \ 3^3 = 27; \ 3^4 = 81 \]
Since 54 is between 27 and 81, \( x \) MUST be a number between 3 and 4.

C. \( 3 < x < 4 \)

**Trap: E. A number between 3 and 4 is also less than 5. But a number less than 5, such as 1 or 2 does not HAVE to be between 3 and 4, so while E. COULD be trust, it isn’t always.**

16. **J. Numbers: LCM Least Common Multiple**

That’s right class, you are now back in fourth grade being tested on multiples. I like to break these numbers into factors:

\[
\begin{align*}
70 &= 7 \times 10 \\
60 &= 6 \times 10 \\
50 &= 5 \times 10
\end{align*}
\]

Our answer has to have common parts, we only need one 10, because each of them has a 10.
\[ (7 \times 6 \times 5) \times 10 = 2,100 \]

J. 2,100

17. **B. Solids**

Volume of a box is \( l \times w \times h \)

Given volume, and two dimensions, find the third.
\[ 81,000 = 45 \times 30 \times h \]

B. 60

18. **J. Circles**

DRAW! If you don’t draw this one out, you are making a mistake.
Clockwise: move to the right
Counterclockwise: move to the left.

\[ J, A, C, D, B \]

**Tip:** When you can draw something for a problem, DRAW! Physically moving your pencil uses a different part of your brain

19. **D. Exponents & Roots**

Substitute \( t = 5 \) into \( y = 16(2)^x \)

\[ y = 16(2)^5 = 512 \]

D. 512

**Trick:** Set up your equation and then take a peek at the answer choices. D is the only possible answer since NO power of 2 ends in ZERO! 16 is a power of two, therefore the answer MUST be a power of 2.
You know that 512 is a power of 2. This could be a 10 second problem – you can solve it faster than most students can pick up and enter the equation in their calculator.

20. **J. Quadrilaterals**

DRAW!

\[ k = 3 \]

J. 3
21. **E. Algebraic Operations**

(a + 2b + 3c) − (4a + 6b − 5c) Distribute the negative sign

(a + 2b + 3c) − 4a − 6b + 5c  CLT: Combine like terms

**E.** −3a − 4b + 8c

**Trap:** Be very careful distributing negative signs. This problem does not take long to do, so get it right by being mindful and executing.

22. **G. Trigonometry**

\[ \sin \theta = \frac{\text{Opp}}{\text{Hyp}} \]

\[ \sin \theta = \frac{a}{c} \]

**G:** \( \frac{a}{c} \)

**Tip:** The Trig on the ACT is usually this easy. Practice this problem and gain speed but do it with accuracy.

23. **B. COUNTING, PERMUTATIONS, & COMBINATIONS**

Label the five players A, B, C, D, E and put them in a circle.

1. A can only pass to C or D, pick C
2. C can only pass to E
3. E can only pass to B
4. B can only pass to D
5. D can only pass to A - and thus it takes five passes for the ball to get back to A.

**B. 5th**

24. **H. Linear Functions:** \( y = mx + b \)

\[ y = 0.12x + 3000 \]  the slope is 0.12

Add 0.1 to 0.12

**H. 0.22**

25. **A. Exponents & Roots**

\[-8x^3(7x^6 − 3x^5)\] Distribute, be careful of the negative signs.

When multiplying variables with same base but different exponents, add them.

\[ (−8x^3) \cdot (7x^6) + (−8x^3) \cdot (−3x^5) \]

Always use parenthesis when manipulating terms in Algebraic expressions and equations.

**Tip:** Look at the answer choices. The answer must be two terms, first negative, second positive. That leaves only A and C. You can be down to 50/50 just by looking at the signs.

\[ (−8 \cdot 7 \cdot x^{3+6}) + (−8 \cdot −3 \cdot x^{3+5}) \]

**A.** −56x^9 + 24x^8

**E** is just wrong; it is for someone who clearly does not understand exponents.

26. **G. ABS Absolute Value**

This one you can type right into your calculator

\[ −3|−6 + 8| \]

\[ −3|2| \]

**G. − 6**

27. **B. Triangles**

Solve for \( BC \), a 3-4-5 right triangle. AEC is a similar triangle \( AC = BC \cdot 4 \)

so \( AE = BD \cdot 4 = 3 \cdot 4 = 12 \)

**B. 12**
28. H. Functions \( f(x) \)

This is a linear function because it says "constant rate."

The easiest thing to do is substitute \( t=0; y=14 \) into each answer. Eliminate G, J, K

Then substitute \( t = 1; y = 19 \)

\[
F: (19) = (1) + 14 - \text{wrong}
\]

\[
H: (19) = 5(1) + 14 - \text{correct}
\]

**H. \( y = 5t + 14 \)**

29. E. Inequalities

\[
6x + 12 > 7x - 35 \quad \text{* Simplify}
\]

\[
12 + 35 > 7x - 6x
\]

\( 47 > x \)

**E. \( x < 47 \)**

30. K. Coordinate

![Diagram of coordinates with points labeled F, G, J, K, and a point labeled (2,0)]

Only K is 3 units away from \((2,0)\)

**K. \((5,0)\)**

31. F. Triangles

*It does not get much easier than this:*

\[
A^2 + B^2 = C^2
\]

\[
y^2 = x^2 + 4^2
\]

\[
y = \sqrt{x^2 + 16}
\]

**E. \( \sqrt{x^2 + 16} \)**

32. G. Probability

Change \( \frac{12}{32} \) to \( \frac{3}{5} \)

1. BACKSOLVE:

Add answers to both the numerator and denominator and reduce.

**Tip: the new fraction’s denominator is 5, therefore the right combination MUST be a multiple of 5; ending in 5 or 10. J & K can be eliminated.**

\[
F: \frac{12+13}{32+13} = \frac{25}{45} = \frac{5}{9}
\]

\[
G: \frac{12+18}{32+18} = \frac{30}{50} = \frac{3}{5} \text{ correct!}
\]

2. Algebraic Solution:

\[
\frac{12 + x}{32 + x} = \frac{3}{5}
\]

\[
5(12 + x) = 3(32 + x)
\]

\[
60 + 5x = 96 + 3x
\]

\[
2x = 36
\]

\[
x = 18
\]

**G. 18**
33. D. Coordinate Geometry & XY-Plane

4x - 2y = 8
The line is in standard form, to plot on xy-plane, just find the intercepts (zero’s).

4(0) - 2y = 8
y = -4; (0, -4)

4x - 2(0) = 8
x = 2; (2,0)

Plot:

D. I, III, IV only

34. F. Linear Functions

Substitute and solve

y = -5x^2 + 9  Substitute (1,2a)
(2a) = -5(1)^2 + 9
2a = 4; a = 2

F. 2

35. D. Ratio

\[ \frac{1}{2}(S) + \frac{1}{3}(S) + x(S) = S \]

\[ x = \frac{1}{6} \]

Convert to ratio

\[ \frac{1}{2} : \frac{1}{3} : \frac{1}{6} \text{ is equal to } 3 : 2 : 1 \]

D. 3 : 2 : 1

36. F. Equation of Circle

The equation of a circle in the standard \((x, y)\) plane is:

\[(x - h)^2 + (y - k)^2 = r^2\]

Where \((h, k)\) is the center of the circle and \(r\) is the radius.

\[(x - 5)^2 + y^2 = 38\]

\[r^2 = 38; r = \sqrt{38}\]

\[a = 5, b = 0 \text{ center is } (5,0)\]

Tip: Be careful of negative signs for the center of a circle. They move like functions \((x-5)\) moves to the right.

F. Radius = \(\sqrt{38}\) Center = \((5,0)\)

37. B. Multiple Figures

You are asked for the outside perimeter of the of the figure.

2 semicircles make one circle, find circumference

\[ C = \pi d; C = 8\pi \]

Add only two sides of the square, 16

B. 16 + 8\pi
38. G. Multiple Figures

Two "Midpoints" cut a rectangle in half, forming two congruent smaller rectangles.

\[ ABEF \cong EFCD \]

A diagonal of a rectangle cuts the rectangle into equal triangles.

\[ \Delta ABE \cong \Delta AEF \& \Delta EFC \cong \Delta EDC \]

The diagonals of a rectangle intersect at the center of the rectangle and form four congruent triangles.

So each half of the rectangle is split into four equal pieces, one non-shaded piece and three shaded pieces, making a ratio of 1:3.

G. 3:1

Tip: "Midpoints" ALWAYS means cut the thing in half!

39. C. Linear Functions: \( y=mx+b \)

Trick: Read the question carefully. It is ONLY asking about \( x \) coordinates. The \( y \) coordinates are irrelevant.

\[ \text{Midpoint} = \frac{(14-(-4))}{2} \]

\[ -4 + 9 = 5 \]
\[ 14 - 9 = 5 \]

C. 5

40. G. Solids

Area of one face of a cube, side 8 = \( 8^2 = 64 \)

Six sides of a cube \( 6 \times 64 = 384 \)

G. 384

41. B. System of Equations

\[ ay + bx = c \]
\[ ay - bx = c \]

Put both equations into slope-intercept form:

\[ y = -\frac{b}{a}x + \frac{c}{a} \quad \text{* slope} = -\frac{b}{a} \]
\[ y = \frac{b}{a}x + \frac{c}{a} \quad \text{* slope} = \frac{b}{a} \]

I. Parallel Lines – No. Parallel lines have the same slope.

II. Intersecting Lines – Yes. Intersecting lines have different slopes (same \( y \) intercept means that they intersect on the \( x \)-axis).

III. Single Line – No. Single lines have the same slope and the same \( y \) intercept.

B. II only
42. F. 401; Trigonometry

From the angle of 52, the opposite side is \( x \) and the adjacent side is 30.

\[
\tan \theta = \frac{\text{opposite}}{\text{adjacent}} \quad \tan 52 = \frac{x}{30} \quad \text{solve for } x
\]

\[
x = 30 \tan 52
\]

F. 30 \( \tan 52 \)

43. D. Charts & Graphs

Odds – in age: not in age

\((25 - 35) = 42\% \quad \text{(all others)} = 58\%\)

42 : 58 = 21 : 29

D. 21 : 29

44. H. Multiple Figures

“Lines of symmetry” are lines that are when folded each side is a mirror reflection. There are 8 ways to “fold” this figure.

H. 8

45. A. Circles

Diameter = 2, so radius = 1
Area = \( \pi r^2 = 3.14 \)

A. 3.1

46. J. Percent

Diameter = 2, 75% longer

\[
2 \times 1.75 = 3.5
\]

J. 3.50

47. C. Geometry Lines

\( \triangle BAC \) & \( \triangle ACD \) are complimentary angles (opposite interior) and add up to 180.

\( \angle BAC = 82 \quad \angle ACD = 180 - 82 = 98 \)

Each are \( \text{bisected} \)

\[
\angle EAC = \frac{82}{2} = 41
\]

\[
\angle ACE = \frac{98}{2} = 49
\]

\[
\angle AEC = 180 - (41 + 49) = 90
\]

C. 90
48. H. Circles

Tip: When a triangle is in a circle and one vertex is the center and the other two are on the circle, radii form two sides, making it an isosceles triangle – this is a standardized test favorite.

Arc angle is twice the angle of the two chords.

49. B. 210; System of Equations

Infinite solutions means “same line.”
1. Using slope intercept form

\[ y = 2x - 8 \]

\[ y = 2x - \frac{4}{3}a \]

The two lines have the same slope they need to have the same intercept. Therefore:

\[-\frac{4}{3}a = -8 \quad \text{solve for } a\]

B. 6

50-52. Systems of Equations

This is a rare topic, but one that uses simple linear functions and can be understood easily.

50. F. Systems of Equations

The graph shows a horizontal line at \( y = 2 \), as the bottom constraint. Because this is a system of inequalities, the shaded area above the line \( y = 2 \) represents the number of large frames that she can make. Whereas, the \( x \) axis represents number of small frames. For example: The point (2,3), she CAN make 2 small and 3 large. But (3,1) will not be made since it is not shaded.

The \( y \) axis is the number of large frames. Therefore, she makes a minimum of 2 and a maximum of 8.

F. 2

51. C. Rate & Proportion, Percent

“For every hour” = $3

Find the number of hours worked.

Large = (3hrs)(4L) = 12hrs
Small = (2hrs)(2S) = 4hrs
Total = 12 + 4 = 16hrs

($3/hr)(16hrs) = $48 donated

Now find profit

30(2) + 70(4) = 340

Percent donated: 48/340 = .1411

C. 14%

52. J. Systems of Equations

A maximum comes at the vertex of the polygon formed by the inequalities in linear programming (a system of linear inequalities).

The maximum profit is when she makes only 8 large frames.

(8 frames)*($70) = $560

J. $560

53. E. Matrix

Don’t sweat matrix problems, the ACT is very nice and tells you exactly what to do.

\[ ab - cb \]

\[ x * x - 8x = -16 \quad \text{solve for } x \]

\[ x^2 - 8x + 16 = 0 \quad \text{perfect square = factor!} \]

\[ (x - 4)(x - 4) \text{ or } (x - 4)^2 = 0 \]

\[ x = 4 \]

E. 4.
54. **K. Algebraic Operations**

This problem has nothing to do with interest! It is simply solving for one variable in terms of another. The ACT is just trying to scare some students off, if they get this far.

\[ A = P(1 + 0.01i)^n \]

*to solve for P, simply divide by what is not P

\[
\frac{A}{(1 + 0.01i)^n} = \frac{P(1 + 0.01i)^n}{(1 + 0.01i)^n}
\]

\[ P = \frac{A}{(1 + 0.01i)^n} \]

\[ K. \quad \frac{A}{(1 + 0.01i)^n} \]

55. **C. Solving Inequalities**

"Must be true" – means that you only need to find one example that proves it wrong. Pick some numbers and get to work.

A. \[ \frac{x}{y} > 1 \]

Never true: A positive number divided by a negative number is always negative.

B. \[ |x|^2 > |y| \]

This statement could be true, but is not always true.

Make \( x = 2 \) & \( y = -200 \).

\[ 4 > 200 \]

C. \[ \frac{x}{3} - 5 > \frac{y}{3} - 5 \]

Simplify: add 5 to both sides and then multiply by 3

\[ x > y \quad \text{A positive number is always greater than a negative number.} \]

**This must be true.**

D. \[ x^2 + 1 > y^2 + 1 \]

Use the same logic as B. Make \( x = 2 \) & \( y = -200 \).

E. \[ x^{-2} > y^{-2} \]

Use the same logic as B. Make \( x = 2 \) & \( y = -200 \)

56. **J. Triangles**

Fun little problem. Here you have to find the height of the triangles (also called altitude). And then compare triangles.

Set \( x \) as the base of both triangles:

Then the trick is to see that the height is the same. The height, \( h \), must be congruent using Angle-Side-Angle (yup, those old lovely Geometry triangle proofs).

57. **E. Trigonometry**

**Tip:** Memorize the Law of Sines and the Law of Cosines. Look at the examples of when each are used on the ACT. Generally, the ACT only asks you to set up the equation, not solve it.

Here it is the Law of Cosines that must be used because we have the length of two sides and the angle in between them.

\[ BC^2 = 12^2 + 18^2 - 2(12)(18) \cos(40^\circ) \]

\[ E. \quad BC = \sqrt{12^2 + 18^2 - 2(12)(18) \cos(40^\circ)} \]
58. G. Sequence

For most sequences, I like to draw a picture of the number blanks.

From 8 to 13 moving 4 spaces is $\frac{(13-8)}{4} = \frac{5}{4}$ Each space is $\frac{5}{4}$

Now find the first 4 terms.

Add them up: $1.75 + 3 + 4.25 + 5.5 = 14.5$

G. 14.5

59. C. Quadratics & Parabolas

Tip: When a quadratic equation is equal to zero, you are solving for the “roots” – otherwise known as “factoring.” When there is only one solution or root the quadratic is a “perfect square” with its vertex on the x axis.

If the only solution is $x = -3$ then the quadratic is $(x + 3)^2 = 0$

Expand: $x^2 + 6x + 9 = 0; m = 6$

C. 6

60. F. ABS Absolute Value

Tip: Absolute value is used to measure distance from zero. “5 units from -3” means numbers that are a distance of 5.

The solution set is (-8,2); Either put the solution into the equations or solve the equations:

$|x + 3| = 5$

$(x + 3) = -5 \; & \; (x + 3) = 5$

$x = -8 \; & \; x = 2$

F. $|x + 3| = 5$