

SAT and ACT Math Guide  
With Cross References to Official SAT and ACT Practice Tests

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## 1 Start Here!

### How to use this guide

This guide to SAT and ACT math covers all math content tested on the SAT and the ACT. In addition, each section cross references with problems on official SAT and ACT practice tests, which are available for free. This provides significant benefit because it means you will receive instruction on all content areas and only be doing official practice problems and don't need to go out and buy any resources. It also means you can do numerous practice problems from a specific content area and master it before moving on to different kinds of problems.

The guide is set up somewhat sequentially. However, you will likely have a firm grasp of some of the content. Therefore, each section is designed to stand alone as much as possible. If you know specific content areas that you struggle with, you should begin with the content and practice problems in those sections.

Finally, please feel free to pass this guide on to anyone you know who may need some guidance studying for the math sections of the SAT and the ACT!

### Where to get the tests

The most valuable part of this guide is the references to official SAT and ACT practice tests for each content area. You should get these practice tests and use them with each section. The tests are available for free at the links below.

SAT 1	<a href="http://www.mattboutte.com/SAT1">www.mattboutte.com/SAT1</a> <a href="http://www.mattboutte.com/SAT1Answers">www.mattboutte.com/SAT1Answers</a> <a href="http://www.mattboutte.com/SAT1Explanations">www.mattboutte.com/SAT1Explanations</a>
SAT 2	<a href="http://www.mattboutte.com/SAT2">www.mattboutte.com/SAT2</a> <a href="http://www.mattboutte.com/SAT2Answers">www.mattboutte.com/SAT2Answers</a> <a href="http://www.mattboutte.com/SAT2Explanations">www.mattboutte.com/SAT2Explanations</a>
SAT 3	<a href="http://www.mattboutte.com/SAT3">www.mattboutte.com/SAT3</a> <a href="http://www.mattboutte.com/SAT3Answers">www.mattboutte.com/SAT3Answers</a> <a href="http://www.mattboutte.com/SAT3Explanations">www.mattboutte.com/SAT3Explanations</a>
SAT 4	<a href="http://www.mattboutte.com/SAT4">www.mattboutte.com/SAT4</a> <a href="http://www.mattboutte.com/SAT4Answers">www.mattboutte.com/SAT4Answers</a> <a href="http://www.mattboutte.com/SAT4Explanations">www.mattboutte.com/SAT4Explanations</a>
SAT 5	<a href="http://www.mattboutte.com/SAT5">www.mattboutte.com/SAT5</a> <a href="http://www.mattboutte.com/SAT5Answers">www.mattboutte.com/SAT5Answers</a> <a href="http://www.mattboutte.com/SAT5Explanations">www.mattboutte.com/SAT5Explanations</a>
SAT 6	<a href="http://www.mattboutte.com/SAT6">www.mattboutte.com/SAT6</a> <a href="http://www.mattboutte.com/SAT6Answers">www.mattboutte.com/SAT6Answers</a> <a href="http://www.mattboutte.com/SAT6Explanations">www.mattboutte.com/SAT6Explanations</a>
SAT 7	<a href="http://www.mattboutte.com/SAT7">www.mattboutte.com/SAT7</a> <a href="http://www.mattboutte.com/SAT7Answers">www.mattboutte.com/SAT7Answers</a> <a href="http://www.mattboutte.com/SAT7Explanations">www.mattboutte.com/SAT7Explanations</a>
SAT 8	<a href="http://www.mattboutte.com/SAT8">www.mattboutte.com/SAT8</a> <a href="http://www.mattboutte.com/SAT8Answers">www.mattboutte.com/SAT8Answers</a> <a href="http://www.mattboutte.com/SAT8Explanations">www.mattboutte.com/SAT8Explanations</a>
ACT 2005-06	<a href="http://www.mattboutte.com/ACT2005">www.mattboutte.com/ACT2005</a>
ACT 2008-09	<a href="http://www.mattboutte.com/ACT2008">www.mattboutte.com/ACT2008</a>
ACT 2011-12	<a href="http://www.mattboutte.com/ACT2011">www.mattboutte.com/ACT2011</a>
ACT 2014-15	<a href="http://www.mattboutte.com/ACT2014">www.mattboutte.com/ACT2014</a>
ACT 2016-17	<a href="http://www.mattboutte.com/ACT2016">www.mattboutte.com/ACT2016</a>

(The ACT frequently puts out the same practice test several years in a row, which is why some years are skipped.)

## How to get a print copy of this guide

If you would prefer to have a print copy of this guide (or to save your printer some ink!), you can buy a copy of this guide at the link below.

[www.mattboutte.com/buy](http://www.mattboutte.com/buy)

## Connect with Matt

Have a question or comment? Or just want to follow Matt as he travels the world? Connect with him on your favor channel below!

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## Get tutoring

Have you looked over this guide and still need some help? Matt would love to help you out! He does all his tutoring online and works with students all over the world (and travels all over the world). He has a great online set up that will make working together and mastering these tests as pain free as possible. You can book appointments with him at the link below.

[www.mattboutte.com/tutor](http://www.mattboutte.com/tutor)

## Want to leave a tip?

This guide took a lot of time and effort to create and I've been told that it's helpful. I'm happy to provide it to anyone and everyone for free. If you find it helpful and would like to leave a tip, you can send me a few bucks on Venmo or PayPal.

**Venmo** – @mattboutte

**Web** – [www.mattboutte.com/donate](http://www.mattboutte.com/donate)

## 2 Basic Algebra

### 2.1 Basic Tools

#### 2.1.1 Keywords

There are lots of keywords that can help with interpreting word problems. Most of these are pretty intuitive. However, two are worth noting because they occur so frequently and regularly throw students off.

**of** – always means multiply

**per** – always means divide

### 2.1.2 Solving Basic Algebraic Expressions

Solving basic algebraic expressions requires working towards getting a variable alone. To do this, inverse operations must be undone in reverse order of operations (PEMDAS).

$$3x^2 + 4 = 79$$

Ordinarily, the order of operations would tell us to do the exponent, then the multiplication, and finally the addition. But because we are solving for  $x$ , we do these in reverse order.

$$3x^2 = 75$$

$$x^2 = 25$$

$$x = \pm 5$$

A word about fractions. If you are undoing a fraction that is being multiplied by a variable, the inverse operation would be to divide by the fraction. This will work, but it is easier to simply multiply by the reciprocal of the fraction.

By dividing:

$$\frac{3}{4}x = 15$$

$$\frac{\frac{3}{4}x}{\frac{3}{4}} = \frac{15}{\frac{3}{4}}$$

By multiplying by the reciprocal:

$$\frac{3}{4}x = 15$$

$$\frac{4}{3}\left(\frac{3}{4}x\right) = (15)\frac{4}{3}$$

### 2.1.3 Combining Like Terms

Combining like terms is a basic mathematical operation. However, it can be visual confusing. It is therefore important to create visual cues for yourself, such as crossing out terms once you have used them.

To combine like terms, simply identify terms that have the same combination of variables and exponents and then combine their coefficients.

$$\begin{array}{r} \underline{5x^2y} + \underline{12xy^2} - \underline{6xy} - \underline{7x^2y} + \underline{xy^2} + \underline{5xy} \\ -\underline{2x^2y} + \underline{13xy^2} - \underline{xy} \end{array}$$

### 2.1.4 Fraction Busting

There are numerous types of problems that involve fractions that can be made significantly easier by eliminating the fractions. This can be done by multiplying *all* terms by the common denominator (also known as “fraction busting”).

For example, if we have the following equation

$$\frac{x}{2} + \frac{x+3}{3} = \frac{5}{x}$$

we can eliminate all the fractions by multiplying by a term to cancel with all the denominators. 6 will cancel with the 2 and the 3, and we will need an  $x$  as well to cancel with the  $x$  in the denominator.

$$\begin{aligned}6x\left(\frac{x}{2} + \frac{x+3}{3} = \frac{5}{x}\right) \\6x\left(\frac{x}{2}\right) + 6x\left(\frac{x+3}{3}\right) &= 6x\left(\frac{5}{x}\right) \\3x(x) + 2x(x+3) &= 6(5) \\3x^2 + 2x^2 + 6x &= 30\end{aligned}$$

This equation will be significantly easier to solve than the one involving fractions.

As mentioned, fraction busting can be used in several different contexts. For example, it can be used to make solving systems of equations easier.

$$\begin{aligned}\frac{x}{3} &= 2y - 1 \\2x - y &= 10\end{aligned}$$

This system would be a lot easier to solve if the first question didn't involve fractions. We can make things a lot easier if we multiply it by 3.

$$\begin{aligned}3\left(\frac{x}{3} = 2y - 1\right) \\3\left(\frac{x}{3}\right) = 3(2y) + 3(-1) \\x = 6y - 3\end{aligned}$$

This will make solving the system of equations a lot easier.

### 2.1.5 Cross Multiplying

Cross multiplying is a powerful tool for eliminating fractions whenever you have a single fraction equal to another single fraction. (Keep in mind that any whole number is a fraction.) In fact, cross multiplying should be used whenever you have a single fraction equal to another single fraction.



Cross multiplying is done by simply multiplying the numerator (top) of one fraction by the denominator (bottom) of the other fraction and setting it equal to the denominator (bottom) of the first fraction times the numerator (top) of the second fraction.

$$\begin{aligned}\frac{x}{4} &= \frac{x+2}{6} \\ 6x &= 4(x+2) \\ 6x &= 4x+8 \\ 2x &= 8 \\ x &= 4\end{aligned}$$

An important tip to keep in mind is that all numbers are fractions, even if they don't look like one. A number (such as 5) or a variable (such as  $x$ ) or an expression (such as  $5y-3$ ) can be written as a fraction by placing them over one. This can easily turn a fraction problem into a cross multiplying problem, and thus eliminate the fractions.

$$\begin{aligned}7 &= \frac{21}{x-4} \\ \frac{7}{1} &= \frac{21}{x-4} \\ 7(x-4) &= 21 \\ 7x-28 &= 21 \\ 7x &= 49 \\ x &= 7\end{aligned}$$

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 3	1	SAT 1 Sec. 3	5	SAT 1 Sec. 3	8
SAT 1 Sec. 4	2	SAT 1 Sec. 4	10	SAT 1 Sec. 4	35
SAT 2 Sec. 3	1	SAT 2 Sec. 3	8	SAT 2 Sec. 4	3
SAT 2 Sec. 4	3	SAT 2 Sec. 4	6	SAT 2 Sec. 4	32
SAT 2 Sec. 4	33	SAT 2 Sec. 4	37	SAT 2 Sec. 4	38
SAT 3 Sec. 3	1	SAT 3 Sec. 3	2	SAT 3 Sec. 3	5
SAT 3 Sec. 3	17	SAT 3 Sec. 4	6	SAT 3 Sec. 4	7
SAT 3 Sec. 4	10	SAT 3 Sec. 4	25	SAT 3 Sec. 4	33
SAT 3 Sec. 4	37	SAT 4 Sec. 3	5	SAT 4 Sec. 3	6
SAT 4 Sec. 3	10	SAT 4 Sec. 4	5	SAT 4 Sec. 4	31
SAT 5 Sec. 3	6	SAT 5 Sec. 3	17	SAT 5 Sec. 3	19
SAT 5 Sec. 4	4	SAT 5 Sec. 4	5	SAT 5 Sec. 4	8
SAT 5 Sec. 4	33	SAT 6 Sec. 3	15	SAT 6 Sec. 3	17
SAT 6 Sec. 4	1	SAT 6 Sec. 4	9	SAT 6 Sec. 4	32
SAT 6 Sec. 4	33	SAT 7 Sec. 3	2	SAT 7 Sec. 3	16
SAT 7 Sec. 3	20	SAT 7 Sec. 4	2	SAT 7 Sec. 4	23
SAT 7 Sec. 4	23	SAT 8 Sec. 3	1	SAT 8 Sec. 3	12
SAT 8 Sec. 4	1	SAT 8 Sec. 4	9	SAT 8 Sec. 4	32
SAT 8 Sec. 4	38	ACT 2005-06	4	ACT 2005-06	11
ACT 2005-06	15	ACT 2005-06	18	ACT 2005-06	40
ACT 2005-06	41	ACT 2005-06	42	ACT 2005-06	43
ACT 2005-06	44	ACT 2005-06	48	ACT 2008-09	2
ACT 2008-09	6	ACT 2008-09	11	ACT 2008-09	13
ACT 2008-09	21	ACT 2008-09	29	ACT 2008-09	48
ACT 2008-09	52	ACT 2011-12	4	ACT 2011-12	6
ACT 2011-12	8	ACT 2011-12	11	ACT 2011-12	26
ACT 2011-12	31	ACT 2014-15	2	ACT 2014-15	5
ACT 2014-15	7	ACT 2014-15	9	ACT 2014-15	19
ACT 2014-15	21	ACT 2014-15	25	ACT 2016-17	23
ACT 2016-17	48	SAT 5 Sec. 4	21	ACT 2005-06	1
ACT 2005-06	7	ACT 2005-06	39	ACT 2008-09	5
ACT 2008-09	20	ACT 2008-09	34	ACT 2011-12	56
ACT 2014-15	16	ACT 2014-15	53	ACT 2016-17	18
ACT 2016-17	19	ACT 2016-17	45	ACT 2016-17	56

## 2.2 Mid Point and Distance Formulas

**Mid Point Formula** The mid-point is simply the middle point between two other points. Thus, it is found simply by averaging the x values of the two points and the y values of the two points.

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Distance Formula** The distance formula is simply the Pythagorean Theorem. If you can't remember it, just sketch the two point, form a triangle, and then perform the Pythagorean

Theorem.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Test	Number	Test	Number	Test	Number
ACT 2005-06	17	ACT 2008-09	37	ACT 20011-12	9
ACT 2011-12	42	ACT 2011-12	48	ACT 2014-15	39
ACT 2016-17	32				

## 2.3 Proportions

Proportions are two fractions that equal one another. The first fraction represents one scenario and the second fraction represents a second scenario. The numerators (tops) of each fraction measure one quantity while the denominators (bottoms) of each fraction measure another quantity.

After setting up the proportions and setting them equal to each other, the unknown value can easily be solved for by cross multiplying.

If 24 magazines stack to be 5 inches tall, how tall will a stack of 40 magazines be?

$$\begin{aligned} \frac{5 \text{ inches}}{24 \text{ magazines}} &= \frac{x \text{ inches}}{40 \text{ magazines}} \\ 24x &= 200 \\ x &= 8\frac{1}{3} \text{ inches} \end{aligned}$$

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 4	6	SAT 2 Sec. 4	2	SAT 3 Sec. 4	9
SAT 3 Sec. 4	11	SAT 3 Sec. 4	19	SAT 4 Sec. 3	20
SAT 5 Sec. 3	3	SAT 5 Sec. 4	31	SAT 5 Sec. 4	32
SAT 7 Sec. 4	15	SAT 7 Sec. 4	25	SAT 8 Sec. 4	31
ACT 2005-06	3	ACT 2005-06	32	ACT 2005-06	51
ACT 2011-12	18	ACT 2016-17	3	ACT 2016-17	33
ACT 2016-17	51	ACT 2016-17	54	SAT 6 Sec. 4	26

## 2.4 Unit Conversion

Unit conversion is the tool for changing from one unit to another. This is done by first setting up a fraction that will cause the undesired units to cancel out and leave the desired units. After this is set up, the numbers necessary to make the conversation are inserted into the fraction.

If we desire to convert 3 years into days, the fraction would be set up with units as follows.

$$3 \text{ years} \cdot \frac{\text{days}}{\text{years}}$$

Then we put the appropriate numbers in the converter fraction.

$$3 \text{ years} \cdot \frac{365 \text{ days}}{1 \text{ years}}$$

Finally, we cancel the units and perform the necessary arithmetic.

$$3 \text{ years} \cdot \frac{365 \text{ days}}{1 \text{ years}} = 3 \cdot 365 \text{ days} = 1095 \text{ days}$$

This process can be repeated as many times as necessary to convert from years to seconds or from miles per hour to feet per second, for example.

Some unit related formulas are important to know.

$$\text{velocity} = \frac{\text{change in distance}}{\text{change in time}}$$

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{change in time}}$$

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 4	6	SAT 1 Sec. 4	34	SAT 2 Sec. 4	11
SAT 2 Sec. 4	15	SAT 2 Sec. 4	31	SAT 3 Sec. 4	14
SAT 4 Sec. 4	3	SAT 4 Sec. 4	18	SAT 4 Sec. 4	33
SAT 5 Sec. 4	9	SAT 7 Sec. 4	3	SAT 7 Sec. 4	31
SAT 8 Sec. 4	11	SAT 8 Sec. 4	15	SAT 8 Sec. 4	23
SAT 8 Sec. 4	34	SAT 8 Sec. 4	37	ACT 2011-12	40
SAT 5 Sec. 4	37	SAT 6 Sec. 4	29	ACT 2008-09	19
ACT 2016-17	16				

## 2.5 Functions

Functions are simply another way of representing equations such as

$$y = x^2 - 7x + 12$$

In function notation, this would be

$$f(x) = x^2 - 7x + 12$$

As such, functions themselves are easy to work with. But people get confused by the notation. You just need to keep two things straight in your mind.

- $f(3)$  means plug 3 in for  $x$  because the  $x$  in  $f(x)$  has been replaced by a 3
- $f(x) = 7$  means set the whole equation equal to 7 because  $f(x)$  is the “name” of the function

There is one more piece of information you need to keep in mind with functions. You can plug *anything* into a function. For example, using the function  $f(x)$  above, we can plug a heart into it.

$$f(x) = x^2 - 7x + 12$$

$$f(\heartsuit) = \heartsuit^2 - 7\heartsuit + 12$$

This may look odd, but there is nothing mathematically wrong with it. The key idea is that wherever there was an  $x$ , you replace it with a  $\heartsuit$ .

This is important when dealing with composite functions. A composite function is when one function is plugged into another function. This is allowed because you can plug *anything* into a function. The process is the exact same as plugging the heart in – anywhere there was an  $x$ , you replace it. If we are given the functions

$$f(x) = -3x + 5$$

$$g(x) = x^2 + 1$$

and are asked to find  $f(g(x))$ , this simply means that we plug  $g(x)$  into  $f(x)$ . That is, we replace every  $x$  in  $f(x)$  with  $g(x)$ .

$$f(g(x)) = -3(g(x)) + 5$$

$$= -3(x^2 + 1) + 5$$

$$= -3x^2 - 3 + 5$$

$$= -3x^2 + 2$$

When graphing functions, if one function is less than the other, this means that its graph is below the other function’s graph.

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 3	10	SAT 1 Sec. 4	17	SAT 2 Sec. 4	10
SAT 2 Sec. 4	26	SAT 3 Sec. 4	16	SAT 4 Sec. 3	4
SAT 5 Sec. 4	34	SAT 6 Sec. 3	8	SAT 7 Sec. 3	5
SAT 8 Sec. 3	15	SAT 8 Sec. 4	30	ACT 2005-06	19
ACT 2008-09	33	ACT 2008-09	53	ACT 2008-09	57
ACT 2008-09	59	ACT 2011-12	32	ACT 2011-12	50
ACT 2014-15	14	ACT 2014-15	34	ACT 2016-17	5
ACT 2016-17	42				

## 2.6 Complex Fractions

A complex fraction is a fraction made up of two fractions. That is, there is a fraction in the numerator (top) and a fraction in the denominator (bottom).

$$\frac{\frac{5}{x}}{\frac{4x}{15}}$$

There are two steps to handling a complex fraction. Most students remember the second, but not the first.

- Obtain a single fraction in the numerator (top) and a single fraction in the denominator (bottom).
- Invert (flip) the fraction in the denominator (bottom) and multiply it by the fraction in the numerator (top).

$$\frac{\frac{2}{x} + \frac{1}{3}}{\frac{1}{2} - \frac{x}{3}}$$

$$\frac{\left(\frac{3}{3}\right)\frac{2}{x} + \frac{1}{3}\left(\frac{x}{x}\right)}{\left(\frac{3}{3}\right)\frac{1}{2} - \frac{x}{3}\left(\frac{2}{2}\right)}$$

$$\frac{\frac{6}{3x} + \frac{x}{3x}}{\frac{3}{6} - \frac{2x}{6}}$$

$$\frac{\frac{6+x}{3x}}{\frac{3-2x}{6}}$$

$$\frac{6+x}{3x} \cdot \frac{6}{3-2x}$$

$$\frac{6(x+6)}{3x(-2x+3)}$$

$$\frac{2(x+6)}{x(-2x+3)}$$

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 3	13				

## 2.7 Exponent and Radical Rules

There are a number of exponent and radical rules that are tested. You should be comfortable with all of these rules. This means being comfortable using them in situations that you may not have ever seen before. Each of the rules works in two directions; being comfortable with the rules requires being able to use the rules in both directions. The rules are always the same, but they can be tested in novel ways.

**Product Rule**

If the base of two terms is the same and they are being multiplied, the exponents can be added.

$$x^7 \cdot x^{12} = x^{19}$$
$$3^x \cdot 3^{5y} = 3^{x+5y}$$

**Power Rule**

If a power is applied to multiple terms being multiplied together, the power can be applied to each term individually by multiplying the exponents.

$$(2x^3y^5z^4)^6 = 2^6x^{18}y^{30}z^{24}$$
$$(3a^2b^3c^4)^x = 3^x a^{2x} b^{3x} c^{4x}$$

**Quotient Rule**

If the base of two terms are the same and one is in the numerator (top) and the other is in the denominator (bottom) of a fraction, then they may be combined by subtracting the exponents.

$$\frac{x^9}{x^4} = x^{9-4} = x^5$$
$$\frac{y^2}{y^7} = y^{2-7} = y^{-5}$$
$$\frac{z^{-8}}{z^{-3}} = z^{-8-(-3)} = z^{-5}$$

**Negative Exponent Rule**

If a negative exponent occurs in the numerator (top) of a fraction, it may be removed by moving the term to the denominator (bottom) of the fraction. If a negative exponent occurs in the denominator (bottom) of a fraction, it may be removed by moving the term to the numerator (top) of the fraction.

$$x^{-13} = \frac{1}{x^{13}}$$
$$\frac{1}{y^{-7}} = y^7$$

**Zero Exponent Rule**

Anything to the 0 power automatically equals 1.

$$13^0 = 1$$
$$x^0 = 1$$

### Root of a Fraction

The root of a fraction is simply the same root of the numerator (top) and denominator (bottom) of the fraction.

$$\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$$

### Fractional Exponent and Radical Rule

Fractional exponents can be turned into a radical and a radical can be switched into an exponent. Both forms have their pros and cons. Generally, whenever you're stuck in one form, try switching to the other form.

To switch back and forth:

- The denominator (bottom) of the fractional exponent corresponds with the root of the radical and vice versa.
- The numerator (top) of the fractional exponent corresponds with the power of the radical and vice versa.

$$27^{\frac{2}{3}} = \sqrt[3]{27^2}$$

Note that the 2 can go either inside or outside the radical. Generally, but not always, it is easier to put it on the outside of the radical.

### Getting the same base trick

If two exponentials are equal to each other, then getting them to have the same base makes them easy to solve.

$$\begin{aligned} 4^{x-2} &= 8^{2x+4} \\ (2^2)^{x-2} &= (2^3)^{2x+4} \\ 2^{2x-4} &= 2^{6x+12} \end{aligned}$$

Because the bases are equal, the exponents must now be equal.

$$\begin{aligned} 2x - 4 &= 6x + 12 \\ -4x &= 16 \\ x &= -4 \end{aligned}$$

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 3	14	SAT 2 Sec. 3	7	SAT 3 Sec. 3	3
SAT 5 Sec. 3	12	SAT 6 Sec. 3	16	SAT 7 Sec. 3	11
SAT 8 Sec. 4	13	ACT 2005-06	8	ACT 2008-09	4
ACT 2011-12	22	ACT 2011-12	35	ACT 2011-12	49
ACT 2016-17	22	ACT 2011-12	45		

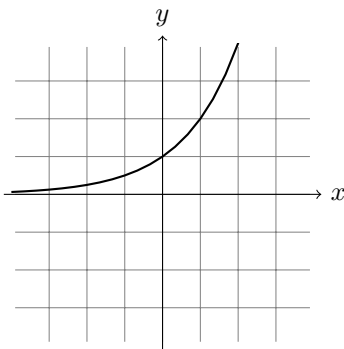


## 2.8 Exponential Problems

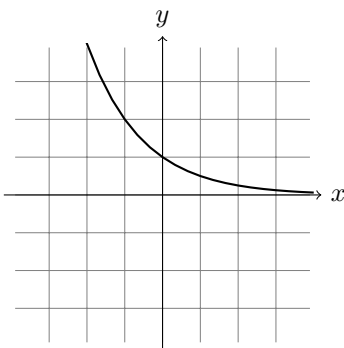
Exponential growth occurs when a quantity grows at a faster and faster rate. This occurs because the quantity is repeatedly multiplied by the same value (as opposed to a linear function, which adds the same amount repeatedly) These equations are of the form

$$y = a \cdot b^x$$

where  $b > 1$



Exponential decay occurs when a quantity shrinks by dividing it by the same amount repeatedly. These equations are of the same form, but  $|b| < 1$ .



Test	Number	Test	Number	Test	Number
SAT 4 Sec. 4	13	SAT 4 Sec. 4	15	SAT 4 Sec. 4	20
SAT 5 Sec. 3	14	SAT 6 Sec. 4	37	ACT 2005-06	49
ACT 2008-09	51	ACT 2008-09	60	ACT 2011-12	30
ACT 2011-12	60	ACT 2014-15	15	ACT 2016-17	7

## 2.9 Radical Problems

When solving an equation involving a radical, we must first isolate the radical alone on one side. Then we raise both sides to whatever power is necessary to eliminate the radical.

$$3\sqrt{x+5} - 2 = 10$$

$$\begin{aligned}
 3\sqrt{x+5} &= 12 \\
 \sqrt{x+5} &= 4 \\
 (\sqrt{x+5})^2 &= 4^2 \\
 x+5 &= 16 \\
 x &= 11
 \end{aligned}$$

When solving radical problems, you must check your solutions by plugging them back into the original equation. If they do not work, they are extraneous and must be excluded.

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 3	20	SAT 2 Sec. 3	5	SAT 4 Sec. 3	9
SAT 5 Sec. 3	5	SAT 6 Sec. 3	9	SAT 8 Sec. 3	7
ACT 2005-06	10	ACT 2011-12	19		

## 2.10 “In Terms Of” Problems

“In terms of” problems are amongst the easiest problems on the SAT and ACT, but are frequently overwhelming to students when they first seem them. These problems present an equation (sometimes an equation that appears to be long and complicated) and then an explanation of what the equation does and what each of the variables means. Then it will ask for one of the variable “in terms of” the other variables. This is the only part that matters (hence the name) – do not spend any time trying to understand the equation or what the variable are.

There are two varieties of “in terms of problems”: ones where the variable you need to isolate (get alone) occurs once, and ones where the variable you need to isolate (get alone) occurs more than once.

### When the variable occurs once

To isolate the variable, simply remove items one by one by doing the opposite operation until the desired variable is alone.

Get  $m$  in terms of  $s$ ,  $t$ , and  $u$ .

$$\begin{aligned}
 t &= s + mu \\
 t - s &= mu \\
 \frac{t - s}{u} &= m
 \end{aligned}$$

### When the variable occurs more than once

To isolate a variable that occurs more than once, follow these steps:

- Each term containing the variable must be put on one side of the equation and each term that doesn't contain the variable must be put on the other side of the equation.
- Factor the variable out of each term on that side of the equation.
- Divide both sides by what is left over to get the variable alone.

Get  $p$  in terms of  $f$ ,  $g$ ,  $h$ , and  $k$ .

$$\begin{aligned}\frac{p-f}{h} &= \frac{p+k}{g} \\ g(p-f) &= h(p+k) \\ gp-gf &= hp+hk \\ gp-hp &= gf+hk \\ p(g-h) &= gf+hk \\ p &= \frac{gf+hk}{g-h}\end{aligned}$$

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 3	7	SAT 1 Sec. 4	9	SAT 2 Sec. 3	12
SAT 2 Sec. 4	22	SAT 3 Sec. 4	13	SAT 5 Sec. 4	10
SAT 6 Sec. 3	7	SAT 6 Sec. 4	17	SAT 7 Sec. 4	19
SAT 7 Sec. 4	26	SAT 8 Sec. 3	3	ACT 2005-06	59
ACT 2014-15	54				

## 2.11 Comparison Problems or Effect On Problems

These problems ask students what effect changing the value of a variable has on the total value of an equation. That is, you compare the value of the original problem to the value of the problem after the variable is changed.

To do these problems, follow these steps:

- Write the original, unchanged equation.
- Rewrite the original equation but with the new value for the changed formula.
- Do any mathematical operations necessary on the new value, but do not combine it with any other numbers present. Instead, move the value towards the outside of the equation.
- Compare the original equation with the new equation.

What is the effect of tripling the radius on the area of a semicircle?

$$A = \frac{1}{2}\pi r^2$$

$$A = \frac{1}{2}\pi(3r)^2$$

$$A = \frac{1}{2}\pi 9r^2$$

$$A = 9\left(\frac{1}{2}\pi r^2\right)$$

This is 9 times the original equation, therefore the effect is to multiply the area of the original semicircle by 9.

Test	Number	Test	Number	Test	Number
SAT 2 Sec. 4	23	SAT 3 Sec. 3	15	SAT 4 Sec. 4	14
SAT 4 Sec. 4	35	SAT 5 Sec. 3	11	SAT 5 Sec. 3	13
ACT 2011-12	29	ACT 2016-17	43	ACT 2016-17	53

## 2.12 Inequalities

### 2.12.1 Solving Inequalities

To solve an inequality, simply treat the inequality sign as an equal sign and solve the equation as you normally would. The only other thing you need to do is turn the inequality around if you multiply or divide both sides by a negative number.

$$-2x - 7 < 5$$

$$-2x < 12$$

$$x > -6$$

If you have an equation with two inequalities in it, you can either break it into two inequalities and treat them as you normally would or you can solve for the variable by doing the same thing to all three sides of the equation (again remember to turn the inequality around if you multiply or divide by a negative).

$$-7 \leq 3x + 2 \leq 17$$

$$-9 \leq 3x \leq 15$$

$$-3 \leq x \leq 5$$

### 2.12.2 Graphing Inequalities

To graph an inequality, the  $y$  must be isolated. The line can then be graphed as if it were an ordinary equal sign, keeping the following points in mind.

- If the equation is less than ( $<$ ) or greater than ( $>$ ), the line will be dashed.

- If the equation is less than or equal to ( $\leq$ ) or greater than or equal ( $\geq$ ), the line will be solid.
- If the equation is greater than ( $>$ ) or greater than or equal to ( $\geq$ ), shade above the line.
- If the equation is less than ( $<$ ) or less than or equal to ( $\leq$ ), shade below the line.

If the equations to be graphed are given to you as an equation with two inequalities in it, simply break the equation into two equations and solve each one for  $y$ .

$$4 < 2x - y \leq 7$$

$$4 < 2x - y \quad 2x - y \leq 7$$

$$-2x + 4 < -y \quad -y \leq -2x + 7$$

$$2x - 4 > y \quad y \geq 2x - 7$$

$$y < 2x - 4 \quad y \geq 2x - 7$$

If the equation is a circle, the following slightly modified rules must be applied.

- If the equation is less than ( $<$ ) or less than or equal to ( $\leq$ ), shade inside the circle.
- If the equation is greater than ( $>$ ) or less than or equal to ( $\geq$ ), shade outside the circle.

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 4	11	SAT 1 Sec. 4	18	SAT 1 Sec. 4	28
SAT 1 Sec. 4	31	SAT 2 Sec. 4	21	SAT 3 Sec. 4	31
SAT 4 Sec. 4	19	SAT 4 Sec. 4	26	SAT 5 Sec. 4	13
SAT 6 Sec. 4	18	SAT 7 Sec. 3	14	SAT 7 Sec. 4	5
SAT 8 Sec. 4	20	ACT 2005-06	22	ACT 2005-06	47
ACT 2008-09	36	ACT 2008-09	49	ACT 2011-12	36
ACT 2011-12	57	ACT 2014-15	29	ACT 2016-17	36
ACT 2016-17	38	ACT 2016-17	49	ACT 2016-17	52
ACT 2014-15	55				

## 2.13 Polynomial Equivalence Problems

If you are given two polynomials that are equal to each other, then the corresponding coefficients must be equal. This is a very useful property in numerous problems.

$$5x^2 - 17x + 13 = 5x^2 + (4 - b)x + 13$$

$$-17 = 4 - b$$

$$b = 21$$

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 3	15	SAT 2 Sec. 3	17	SAT 3 Sec. 3	13
SAT 8 Sec. 3	17				

## 2.14 Absolute Values

### 2.14.1 Solving Absolute Values

There are three steps to solving an absolute value problem. They must be done in order, and you must be sure not to skip any steps.

**Isolate the Absolute Value** First, you must isolate the absolute value alone on one side of the equal sign.

$$3|x - 2| + 5 = 17$$

$$3|x - 2| = 12$$

$$|x - 2| = 4$$

Now the absolute value is isolated.

**Ask Whether the Absolute Value is Possible** Once the absolute value is isolated, it is essential to pause and ask whether the problem is possible. This means asking whether the absolute value could produce the value on the other side of the equal sign. Specifically, an absolute value cannot produce a negative value.

$$|x + 5| = -3$$

This is not possible because an absolute value can never produce a negative value. Therefore, there are no solutions.

If you do not pause and ask whether the absolute value is possible and instead immediately proceed to the next step, you will get the problem wrong. Therefore, you *must* pause and ask it after isolating the absolute value.

**Break the Absolute Value into Two Problems** Once the absolute value has been isolated and you've determined that the problem is possible, you break the problem into two problems – one positive and one negative.

$$|x - 2| = 4$$

$$x - 2 = 4$$

$$x - 2 = -4$$

Then we simply solve each of these problems.

$$x - 2 = 4$$

$$x = 6$$

$$x - 2 = -4$$

$$x = -2$$

### 2.14.2 Creating Absolute Values

Another way of thinking about an absolute value problem is as the two points that are equidistant from a central point. For example, consider the following absolute value problem.

$$|x - 5| = 2$$

$$x - 5 = 2$$

$$x = 7$$

$$x - 5 = -2$$

$$x = 3$$

The two solutions (7 and 3) are each 2 units away from 5 (the central value). Note that these numbers correspond with the two values in the original absolute value problem.

- The 5 (the central value) is the opposite of the number inside the absolute value.
- The 2 (the distance away from the central value) is the number on the other side of the equal sign.

Thus, we can answer a question such as this: *Find the absolute value that represents the two points 6 units away from -2.*

$$|x + 2| = 6$$

because 2 is the opposite of the central value (-2) and we want two point 6 units away from this value.

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 4	8	SAT 4 Sec. 3	1	SAT 6 Sec. 4	28
ACT 2005-06	9	ACT 2005-06	21	ACT 2005-06	53
ACT 2008-09	22	ACT 2008-09	42	ACT 2011-12	1
ACT 2011-12	46	ACT 2014-15	26	ACT 2014-15	60
ACT 2016-17	58				

## 3 Percents

### 3.1 Introduction

Percent problems are amongst the easiest problems on the SAT and ACT, yet they are also amongst the most frequently missed problems. The reason for this is that most people were not correctly taught to use percents at an early age. However, the proper way to do percent problems can easily be learned. This is good news because percent problems are very plentiful.

We cannot do percent problems with percents. Thus the first step in any percent problem is to turn the percent into a decimal. This is done by moving the decimal point twice to the left.

$$87\% = 0.87$$

$$2.5\% = 0.025$$

The second step is to turn this decimal into a multiplier. How the decimal is turned into a multiplier depends on the type of problem.

### 3.2 Increasing Problems

Many percent problems involve a quantity that is increasing. Situations that involve an increasing quantity are frequently in the form of a sales tax, a raise, a tip, growth, interest, or other increasing quantity.

In these types of problems, the multiplier ( $m$ ) is formed as follows

$$m = 1 + p$$

where  $p$  is the percent turned into decimal form. This multiplier  $m$  is then multiplied by the quantity to find the new quantity.

For example, if someone earns \$12.00 per hour and receives a 5% raise, we find the multiplier as follows

$$m = 1 + 0.05 = 1.05$$

and multiply it by the old wage in order to find the new wage:

$$\$12.00 \cdot 1.05 = \$12.60$$

### 3.3 Decreasing Problems

Another common category of percent problems involve a quantity that is decreasing. Situations that involve a decreasing quantity are frequently in the form of a sale, discount, decay, or other decreasing quantity.

In these types of problems, the multiplier ( $m$ ) is formed as follows

$$m = 1 - p$$

where  $p$  is the percent turned into decimal form. This multiplier  $m$  is then multiplied by the quantity to find the new quantity.

For example, if a \$40.00 pair of jeans are 15% off, we find the multiplier as follows

$$m = 1 - 0.15 = 0.85$$

and multiply it by the original price of the jeans in order to find the new price of the jeans:

$$\$40.00 \cdot 0.85 = \$34.00$$



### 3.4 “Percent Of” Problems

The easiest type of percent problems are “percent of” problems. In these problems we are looking for some part or portion of the original quantity. In these types of problems, the multiplier ( $m$ ) is simply

$$m = p$$

where  $p$  is the percent turned into decimal form. The multiplier  $m$  is then multiplied by the original quantity to find the new quantity.

For example, if a high school has 1,560 students and 70% of the students are planning on attending the dance, we simply do

$$1,560 \cdot 0.70 = 1,092$$

to find that 1,092 students will attend the dance.

### 3.5 Percent Word Problems

Many percent problems occur in the form of word problems. The trick to these problems is to determine whether it is a percent increasing or percent decreasing problem, give any unknown value a variable, and to carefully walk through the scenario in chronological order.

If you pay \$43.20 for a pair of jeans after an 8% sales tax, what was the sticker price of the jeans?

This is a percent increasing problem and the unknown value is the original price, which we will call  $p$ . Walking through the problem chronologically, the first thing that happened was that the you picked up the pair of jeans, which had price  $p$ . Then an 8% sales tax was applied, so our multiplier will be 1.08. The result is that the final price became \$43.20. Thus we have the equation that can easily be solved for  $p$ .

$$p \cdot 1.08 = 43.20$$

### 3.6 Turning other things into percents

To determine by what percent something has increased or decreased, simply set up an equation chronologically and use the logic of increasing multipliers and decreasing multipliers from above.

If the population of a city has increased from 60,000 to 63,000, what percent has it increased by?

$$60000 \cdot m = 63000$$

$$m = 1.05$$

Thus the population has increased 5%.

If a stock was priced at \$200 per share but is now priced at \$170 per share, what percent has it decreased by?

$$200 \cdot m = 170$$

$$m = 0.85$$

Thus the stock has lost 15% of its value.

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 4	20	SAT 1 Sec. 4	26	SAT 1 Sec. 4	37
SAT 1 Sec. 4	38	SAT 2 Sec. 3	14	SAT 2 Sec. 4	5
SAT 2 Sec. 4	17	SAT 3 Sec. 4	5	SAT 3 Sec. 4	21
SAT 3 Sec. 4	27	SAT 3 Sec. 4	28	SAT 3 Sec. 4	38
SAT 4 Sec. 3	12	SAT 4 Sec. 4	4	SAT 4 Sec. 4	22
SAT 4 Sec. 4	37	SAT 4 Sec. 4	38	SAT 5 Sec. 4	24
SAT 5 Sec. 4	38	SAT 6 Sec. 3	2	SAT 6 Sec. 3	19
SAT 6 Sec. 4	6	SAT 6 Sec. 4	24	SAT 6 Sec. 4	38
SAT 7 Sec. 3	8	SAT 7 Sec.4	10	SAT 7 Sec. 4	12
SAT 7 Sec. 4	27	SAT 7 Sec. 4	30	SAT 8 Sec. 4	22
ACT 2005-06	2	ACT 2008-09	17	ACT 2008-09	47
ACT 2011-12	7	ACT 2011-12	13	ACT 2014-15	46
ACT 2016-17	6	ACT 2016-17	21	ACT 2016-17	25

## 4 Linear Functions

### 4.1 Finding Linear Functions

Linear functions are functions that are straight lines when graphed. When put in slope intercept form

$$y = mx + b$$

where  $m$  is the slope of the line and  $b$  is the  $y$  intercept of the line, they are easily graphed.

#### Finding the Linear Function When the Slope and a Point are Given

If you are given the slope and a point, the equation of the linear function is easily found by plugging the slope in for  $m$  and then plugging the  $x$  and  $y$  values of the point in to solve for  $b$ .

Find the equation of the line with slope  $-2$  and passing through  $(3, -5)$ .

$$m = -2$$

$$y = mx + b$$

$$y = -2x + b$$

Next we plug the point  $(3, -5)$  in for  $x$  and  $y$  to solve for  $b$ .

$$-5 = -2(3) + b$$

$$-5 = -6 + b$$

$$1 = b$$

$$y = -2x + 1$$

### Finding the Linear Function When Two Points are Given

If two points are given and we are asked to find the equation of the linear function passing through the two points, we must start by finding the slope.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Once the slope has been obtained, we proceed exactly as we would in a problem in which the slope and a point are given.

Find the equation of the line passing through  $(-2, 3)$  and  $(4, 6)$ .

$$m = \frac{6 - 3}{4 - (-2)} = \frac{3}{6} = \frac{1}{2}$$

$$y = \frac{1}{2}x + b$$

Now we plug in a point for  $x$  and  $y$  to find  $b$ . Either point will work.

$$6 = \frac{1}{2}(4) + b$$

$$6 = 2 + b$$

$$4 = b$$

$$y = \frac{1}{2}x + 4$$

## 4.2 Interpreting Linear Functions

There are several word problems that ask you to interpret the  $m$  and  $b$  values of a linear function. This is what the slope and  $y$  intercept mean in the context of a word problem:

- **slope ( $m$ ):** the amount the  $y$  variable changes for an increase of one in the  $x$  value
- **$y$  intercept ( $b$ ):** the initial or starting value or quantity of the problem

## 4.3 Graphing Linear Functions

To graph a linear function, simply put it in the form  $y = mx + b$ . First, plot the  $y$ -intercept (from the  $b$  value). Second, use the slope ( $m$ ) to plot a second point, using the rise and run determined by the slope. Then connect the two points with a line.

## 4.4 Parallel and Perpendicular

If two lines are parallel, this means they have the same slope. Thus, the following two equations are parallel.

$$y = -\frac{3}{5}x + 1 \quad y = -\frac{3}{5}x - 4$$

If two lines are perpendicular, this means that their slopes are negative reciprocals of each other. Thus, the following two equations are perpendicular.

$$y = \frac{3}{4}x + 5 \quad y = -\frac{4}{3}x - 1$$

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 3	4	SAT 1 Sec. 3	6	SAT 1 Sec. 3	12
SAT 1 Sec. 4	15	SAT 1 Sec. 4	16	SAT 1 Sec. 4	22
SAT 2 Sec. 3	3	SAT 2 Sec. 3	6	SAT 2 Sec. 4	8
SAT 2 Sec. 4	12	SAT 2 Sec. 4	25	SAT Sec. 4.	28
SAT 2 Sec. 4	35	SAT 3 Sec. 3	8	SAT 3 Sec. 4	4
SAT 3 Sec. 4	8	SAT 3 Sec. 4	17	SAT 3 Sec. 4	26
SAT 4 Sec. 3	2	SAT 4 Sec. 3	7	SAT 4 Sec. 3	8
SAT 4 Sec. 4	8	SAT 4 Sec. 4	17	SAT 4 Sec. 4	27
SAT 4 Sec. 4	32	SAT 5 Sec. 3	1	SAT 5 Sec. 3	8
SAT 5 Sec. 3	16	SAT 5 Sec. 4	2	SAT 5 Sec. 4	7
SAT 5 Sec. 4	11	SAT 5 Sec. 4	16	SAT 5 Sec. 4	17
SAT 5 Sec. 4	18	SAT 5 Sec. 4	23	SAT 5 Sec. 4	28
SAT 6 Sec. 3	1	SAT 6 Sec. 3	5	SAT 6 Sec. 4	2
SAT 6 Sec. 4	4	SAT 6 Sec. 4	5	SAT 6 Sec. 4	13
SAT 6 Sec. 4	14	SAT 6 Sec. 4	15	SAT 6 Sec. 4	25
SAT 6 Sec. 4	35	SAT 7 Sec. 3	1	SAT 7 Sec. 3	6
SAT 7 Sec. 3	19	SAT 7 Sec. 4	17	SAT 7 Sec. 4	28
SAT 7 Sec. 4	32	SAT 8 Sec. 3	2	SAT 8 Sec. 3	13
SAT 8 Sec. 3	19	SAT 8 Sec. 4	2	SAT 8 Sec. 4	7
SAT 8 Sec. 4	21	SAT 8 Sec. 4	25	SAT 8 Sec. 4	27
SAT 8 Sec. 4	29	ACT 2005-06	5	ACT 2005-06	16
ACT 2005-06	27	ACT 2005-06	29	ACT 2005-06	35
ACT 2005-06	56	ACT 2008-09	1	ACT 2008-09	3
ACT 2008-09	12	ACT 2008-09	26	ACT 2008-09	31
ACT 2008-09	40	ACT 2008-09	43	ACT2011-12	2
ACT2011-12	3	ACT2011-12	17	ACT2011-12	23
ACT2011-12	24	ACT2011-12	52	ACT 2014-15	24
ACT 2014-15	28	ACT 2014-15	33	ACT 2014-15	35
ACT 2014-15	58	ACT 2016-17	8	ACT 2016-17	9
ACT 2016-17	11	ACT 2016-17	34	ACT 2016-17	35
ACT 2016-17	39	ACT 2016-17	46		

## 5 Systems of Equations

### 5.1 Definition and Overview

A system of equations is simply a group of equations. To solve a system of equations, you need to have the same number of equations as variables. Generally on the SAT and the ACT, there will be two variables (usually  $x$  and  $y$ ), so you will need two equations to be able to solve the system.

### 5.2 Methods Solving Systems of Equations

There are two methods for solving a system of equations with two equations and two unknowns. Either will work for any given problem. However, problems will be set up to be done more quickly and easily in one form or the other. Therefore students should be comfortable solving systems of equations using both methods and should solve using whichever method the system is set up for.

#### 5.2.1 Substitution

Substitution is used when one variable is alone (or nearly alone) on one side of the equal sign. This variable is then substituted (or “plugged in” to) the second equation.

$$x = 2y - 7$$

$$-2x - 5y = -4$$

$$-2(2y - 7) - 5y = -4$$

$$-4y + 14 - 5y = -4$$

$$-9y = -18$$

$$y = 2$$

Substituting  $y = 2$  into the first equation

$$x = 2(2) - 7$$

$$x = -3$$

Thus the solution is  $x = -3$ ,  $y = 2$ .

#### 5.2.2 Elimination

The other method for solving systems of equations is elimination (also known as cancellation). Elimination is used when there is a column of  $x$  terms, a column of  $y$  terms, a column of the equals signs, and a column of constant terms. One (or both) of the lines is

multiplied by a constant so that either the  $x$  column or the  $y$  column will cancel when the two equations are added (or subtracted).

$$3x - 5y = 13$$

$$4x + 3y = -2$$

Here the  $y$  column will cancel if we multiply the first line by 3 and multiply the second line by 5.

$$3(3x - 5y = 13)$$

$$5(4x + 3y = -2)$$

$$9x - 15y = 39$$

$$20x + 15y = -10$$

The  $y$  column will now cancel

$$9x - 15y = 39$$

$$\underline{20x + 15y = -10}$$

$$29x = 29$$

$$x = 1$$

Substituting  $x = 1$  into the first equation

$$3(1) - 5y = 13$$

$$3 - 5y = 13$$

$$-5y = 10$$

$$y = -2$$

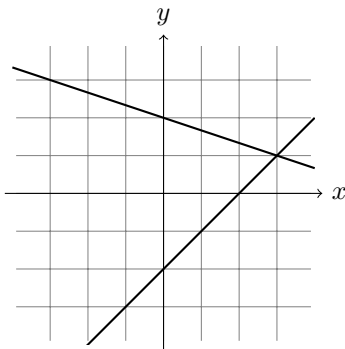
### 5.3 Types of Solutions

Systems of equations have three possible sets of solutions.

- one solution
- zero solutions
- infinitely many solutions

### 5.3.1 One Solution

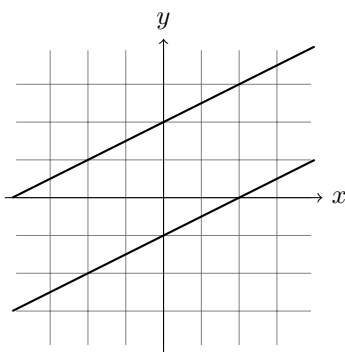
Graphically, this occurs when two lines intersect at a single point.



Algebraically, this occurs when you are able to find numeric values for  $x$  and for  $y$ .

### 5.3.2 Zero Solutions

Graphically, this occurs when two lines are parallel and therefore never intersect.



Algebraically, this occurs when you get an obviously untrue mathematical statement.

$$3x - 4y = 10$$

$$-6x + 8y = -17$$

Multiplying the first line by 2

$$2(3x - 4y = 10)$$

$$-6x + 8y = -17$$

$$6x - 8y = 20$$

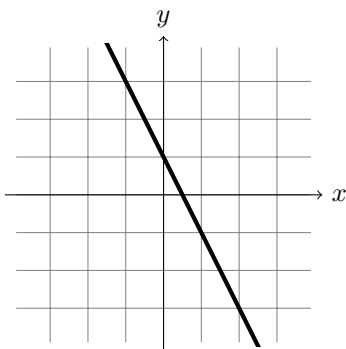
$$-6x + 8y = -17$$

$$\hline 0 \neq 3$$

This is obviously not true, so there are zero solutions.

### 5.3.3 Infinitely many solutions

Graphically, this occurs when two lines are exactly the same and lie on top of each other, thus intersecting at infinitely many points.



Algebraically, this occurs when you get a mathematical statement that is *always* true.

$$5x - 2y = 4$$

$$10x - 4y = 8$$

Multiplying the first line by  $-2$

$$-2(5x - 2y = 4)$$

$$10x - 4y = 8$$

$$-10x + 4y = -8$$

$$\underline{10x - 4y = 8}$$

$$0 = 0$$

This is always true, so there are infinitely many solutions.

## 5.4 Intersections

If you are asked where two lines intersect, this simply means that there is a system of equations. You then simply use the tools for solving systems of equations. The  $x$  and  $y$  values of the solution (or solutions) are simply the point (or points) of intersection of the graphs.

## 5.5 Non-Linear Systems of Equations

Occasionally there are questions with non-linear systems of equations. These are systems of equations in which there are still two equations and two unknowns (generally  $x$  and  $y$ ) but one or more of the variables has a power attached to them. These problems are solved using the exact same methods for solving systems of equations: substitution or elimination. However, these questions are nearly always designed to be solved using substitution.



## 5.6 Turning Word Problems into Systems of Equations

Turning word problems into systems of equations is one of the subjects students struggle with the most on the SAT and the ACT. However, some considerations can be very helpful.

First, clearly define your variables. There will almost always be two variables in these types of questions (occasionally there will only be one variable). Be sure to state exactly what the variable is mathematically. The variable will almost always be the *quantity* of something. For example, if the problem is about apples and bananas, the variables will not be apples and bananas; they will be the *number* of apples and the *number* of bananas.

Second, if you have two variables, you will need two equations. Generally define what your two equations will be about. One will be about the *quantity* of items. The other will be about some other consideration, such as price, dollars, weight, time, etc.

Third, form the equation about the quantity of items. There are two types of quantity equations.

- If you are told the total quantity of items, simply add the two variables and set them equal to this total quantity. For example, if the total number of apples ( $a$ ) and bananas ( $b$ ) is 47

$$a + b = 47$$

- If the two quantities are *compared*, you must think much more carefully about this equation. Students regularly form these equations incorrectly because the English sentence will look somewhat different than the mathematical statement. For example, if there are twice as many apples as bananas

$$a = 2b$$

Student's natural inclination is to put the 2 with the  $a$  because that is where the 2 is in the sentence. However, you must ask which quantity is larger ( $a$ ) and what must be done to compensate for this to make the quantities equal (multiplying  $b$  by 2). Similarly, if we are told there are 12 more apples than bananas

$$a = b + 12$$

because there are fewer bananas and we must add 12 to them to make them equal to the bananas.

Finally, form the equation that isn't about quantity. This is generally easier than the quantity equation.

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 3	9	SAT 1 Sec. 3	11	SAT 1 Sec. 3	18
SAT 1 Sec. 4	4	SAT 1 Sec. 4	19	SAT 1 Sec. 4	32
SAT 2 Sec. 3	2	SAT 2 Sec. 3	9	SAT 2 Sec. 3	20
SAT 2 Sec. 4	1	SAT 2 Sec. 4	9	SAT 2 Sec. 4	34
SAT 3 Sec. 3	4	SAT 3 Sec. 3	6	SAT 3 Sec. 3	9
SAT 3 Sec. 3	10	SAT 2 Sec. 3	19	SAT 3 Sec. 4	18
SAT 3 Sec. 4	22	SAT 3 Sec. 4	24	SAT 3 Sec. 4	30
SAT 3 Sec. 4	36	SAT 4 Sec. 3	3	SAT 4 Sec. 3	19
SAT 4 Sec. 4	1	SAT 4 Sec. 4	2	SAT 4 Sec. 4	6
SAT 4 Sec. 4	16	SAT 4 Sec. 4	30	SAT 4 Sec. 4	34
SAT 5 Sec. 3	7	SAT 5 Sec. 3	9	SAT 5 Sec. 3	15
SAT 5 Sec. 3	18	SAT 5 Sec. 4	6	SAT 5 Sec. 4	12
SAT 5 Sec. 4	35	SAT 6 Sec. 3	6	SAT 6 Sec. 3	16
SAT 6 Sec. 4	10	SAT 6 Sec. 4	11	SAT 6 Sec. 4	19
SAT 6 Sec. 4	31	SAT 7 Sec. 3	3	SAT 7 Sec. 3	9
SAT 7 Sec. 4	11	SAT 7 Sec. 4	16	SAT 7 Sec. 4	20
SAT 7 Sec. 4	33	SAT 8 Sec. 3	5	SAT 8 Sec. 3	6
SAT 8 Sec. 3	10	SAT 8 Sec. 3	18	SAT 8 Sec. 4	6
SAT 8 Sec. 4	12	SAT 8 Sec. 4	33	ACT 2005-06	57
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ACT 2014-15	6	ACT 2014-15	8	ACT 2014-15	10
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ACT 2014-15	52	ACT 2016-17	15	SAT 1 Sec. 3	3
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## 6 Factoring and Related Topics

### 6.1 Factoring

“Factoring” is actually a general terms that describes a number of similar, but distinct, tools. Therefore, it is important to keep each of the different tools distinct and to know when to use each one. In addition, any particular factoring problem may require applying multiple factoring tools or applying the same tool repeatedly.

#### “Pull Out” Factoring

“Pull out” factoring is the most basic type of factoring and also the most overlooked form of factoring. As such, you should *always* ask whether you can perform “pull out” factoring first when approaching a factoring problem. It will always make the problem easier (sometimes dramatically so).

“Pull out” factoring is performed by pulling a common factor out of every term in a polynomial. It can easily be done by asking whether a constant can be pulled out of each term

and then asking whether any variable can be pulled out of each term.

$$12x^3 - 8x^2 + 14x$$

Here, a 2 can be pulled out of each coefficient and an  $x$  can be pulled out as well, so we will pull a  $2x$  out of each term.

$$2x(6x^2 - 4x + 7)$$

### Ordinary Factoring

Ordinary factoring is an essential basic skill for the SAT and ACT, but many students struggle with it. There are numerous different tools you can use to factor. I will demonstrate the box and diamond approach. This method only requires the diamond if  $a = 1$ . If  $a \neq 1$ , it requires both the diamond and the box.

**If  $a$  equals 1** If  $a$  (the coefficient of the  $x^2$  term) is equal to 1, you only need to use the diamond.

$$x^2 + 4x - 12$$

Simply put the  $c$  value in the top of the diamond and the  $b$  value in the bottom of the diamond.

$$\begin{array}{c} \diagup \quad -12 \quad \diagdown \\ \diagdown \quad \quad \quad \diagup \\ \quad \quad \quad 4 \quad \quad \end{array}$$

Then list out in order all the factor pairs of the top number (12) (ignoring the sign).

*1 and 12, 2 and 6, 3 and 4*

Determine which pair can add or subtract to equal the bottom number (4). Here, it is 2 and 6. Put these numbers on the left and right sides of the diamond.

$$\begin{array}{c} \diagup \quad -12 \quad \diagdown \\ \diagdown \quad 2 \quad \diagup \quad 6 \\ \quad \quad \quad 4 \quad \quad \end{array}$$

Next, determine if either of the left or right numbers need a negative sign so that they multiply to be the top number (-12) and add to be the bottom number (4).

$$\begin{array}{c} \diagup \quad -12 \quad \diagdown \\ \diagdown \quad -2 \quad \diagup \quad 6 \\ \quad \quad \quad 4 \quad \quad \end{array}$$

The left and right numbers tell you what the factors will be.

$$x^2 + 4x - 12 = (x - 2)(x + 6)$$

**If  $a$  does not equal 1** If  $a$  (the coefficient of the  $x^2$  term) does not equal 1, you must use the diamond and the box.

$$6x^2 + 5x - 4$$

Multiply the  $x^2$  ( $6x^2$ ) term and the constant term ( $-4$ ) and put them in the top of the diamond. Put the  $x$  term in the bottom of the diamond

$$\begin{array}{c} \diagup \quad \diagdown \\ -24x^2 \\ \diagdown \quad \diagup \\ 5x \end{array}$$

Then list out in order all the factor pairs of the top number (24) (ignoring the sign).

1 and 24, 2 and 12, 3 and 8, 4 and 6

Determine which pair can add or subtract to equal the bottom number (5). Here, it is 3 and 8. Put these numbers with an  $x$  on the left and right sides of the diamond.

$$\begin{array}{c} \diagup \quad \diagdown \\ -24x^2 \\ 3x \quad \quad 8x \\ \diagdown \quad \diagup \\ 5x \end{array}$$

Next, determine if either of the left or right terms need a negative sign so that they multiply to be the top term ( $-24x^2$ ) and add to be the bottom term ( $5x$ ).

$$\begin{array}{c} \diagup \quad \diagdown \\ -24x^2 \\ -3x \quad \quad 8x \\ \diagdown \quad \diagup \\ 5x \end{array}$$

Next, we turn to the box. Put the original  $x^2$  term ( $6x^2$ ) in the top left box and put the original constant term ( $-4$ ) in the bottom right box. The terms from the left and right sides of the diamond go in the remaining two boxes (it doesn't matter which one goes where).

$6x^2$	$-3x$
$8x$	$-4$

Next, pull out factor from each row and column, placing these values along the left and top sides of the box.

	$2x$	$-1$
$3x$	$6x^2$	$-3x$
$4$	$8x$	$-4$

These terms on the left side and top are the factors.

$$6x^2 + 5x - 4 = (3x + 4)(2x - 1)$$

### Factoring by Substitution

Occasionally an equation can be factored more easily by substituting a new variable for the previous variable. This is done so that the equation is of the form  $ax^2 + bx + c$  instead of having different exponents. The following equation is more easily factored by substituting  $u = x^3$ .

$$\begin{aligned} x^6 - 7x^3 + 12 \\ u^2 - 7u + 12 \\ (u - 4)(u - 3) \\ (x^3 - 4)(x^3 - 3) \end{aligned}$$

### Factoring by Grouping

If asked to factor a polynomial of degree three with all four terms present, the only tool available to you for factoring is factoring by grouping. Factoring by grouping is done by doing “pull out” factoring on the first two terms and then doing “pull out” factoring again on the last two terms. This will result in a common factor and the process will be completed by pulling this common factor out. If we are asked to factor the following

$$x^3 + 5x^2 - 9x - 45$$

we know that we will have to factor by grouping because it is a polynomial of degree three with all four terms. We begin by pulling  $x^2$  out of the first two terms and  $-9$  out of the last two terms.

$$x^3 + 5x^2 - 9x - 45$$

$$x^2(x+5) - 9(x+5)$$

Because the  $(x+5)$  is common to both terms, we can pull it out.

$$(x+5)(x^2-9)$$

As mentioned before, the various factoring tools can be applied repeatedly and in various combinations. Here,  $(x^2-9)$  can be factored using ordinary factoring to achieve a final answer.

$$(x+5)(x+3)(x-3)$$

## 6.2 Completing the Square

To complete the square, we take a quadratic and put the  $x^2$  term and the  $x$  term in parentheses with a space with a plus in front of it. We place the constant term outside the parentheses with a space with a negative in front of it.

$$y = x^2 - 6x + 7$$

$$y = (x^2 - 6x + \underline{\quad}) + 7 - \underline{\quad}$$

Then we fill both spaces with the number equal to  $\left(\frac{b}{2}\right)^2$ . Because one is positive and one is negative, they cancel each other out and we have kept the equation balanced.

$$\left(\frac{-6}{2}\right)^2 = 9$$

$$y = (x^2 - 6x + \underline{9}) + 7 - \underline{9}$$

Finally, we factor the terms in the parentheses (which will always be a perfect square) and combine the terms outside the parentheses.

$$y = (x^2 - 6x + 9) - 2$$

$$y = (x-3)^2 - 2$$

When the  $a$  value (the coefficient of  $x^2$ ) is any number other than 1, completing the square requires some additional steps.

$$y = 2x^2 + 20x + 47$$

We begin by creating the parentheses and spaces.

$$y = (2x^2 + 20x + \underline{\quad}) + 47 - \underline{\quad}$$

However, we must now pull the 2 out of the parentheses so that the  $x^2$  is “alone.”

$$y = 2(x^2 + 10x + \underline{\quad}) + 47 - \underline{\quad}$$

Then we fill the first space with the number equal to  $\left(\frac{b}{2}\right)^2$ .

$$\left(\frac{10}{2}\right)^2 = 25$$

$$y = 2(x^2 + 10x + \underline{25}) + 47 - \underline{\quad}$$

We have just added a 25, but its value is actually 50 since it is being multiplied by the 2 in front of the parentheses. So we must put 50 in the second blank space.

$$y = 2(x^2 + 10x + \underline{25}) + 47 - \underline{50}$$

Now we can complete the problem by factoring and combining the terms outside the parentheses.

$$y = 2(x + 5)^2 - 3$$

### 6.3 Polynomial Division

Polynomial division problems are presented as follows. The key is to recognize that this is simply asking for polynomial division to be done.

$$\frac{3x^2 + 2x + 4}{x - 1}$$

The most surefire way to do these problems is with polynomial long division.

$$x - 1 \overline{) 3x^2 + 2x + 4}$$

To do this, we ask what  $x$  must be multiplied by to equal  $3x^2$  – namely,  $3x$ .

$$x - 1 \overline{) 3x^2 + 2x + 4} \quad \begin{array}{r} 3x \\ \hline \end{array}$$

$3x$  is then multiplied by  $x - 1$  and subtracted and then the 4 is brought down.

$$x - 1 \overline{) 3x^2 + 2x + 4} \quad \begin{array}{r} 3x \\ \hline - 3x^2 + 3x \\ \hline 5x + 4 \end{array}$$

Then this process is repeated.

$$x - 1 \overline{) 3x^2 + 2x + 4} \quad \begin{array}{r} 3x + 5 \\ \hline - 3x^2 + 3x \\ \hline 5x + 4 \\ - 5x + 5 \\ \hline 9 \end{array}$$

There are no more terms to bring down, so the remainder is 9 and the answer is as follows:

$$\frac{3x^2 + 2x + 4}{x - 1} = 3x + 5 + \frac{9}{x - 1}$$

## 6.4 Undefined Rational Functions

A rational function is undefined if the denominator (bottom) of the fraction is equal to zero. This frequently requires factoring the denominator (bottom) and setting it equal to zero.

$$y = \frac{1}{x^2 - 8x + 12}$$

$$y = \frac{1}{(x - 6)(x - 2)}$$

Therefore, the function is undefined if  $x = 6$  or  $x = 2$ .

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SAT 1 Sec. 4	36	SAT 2 Sec. 3	4	SAT 2 Sec. 3	15
SAT 3 Sec. 3	13	SAT 3 Sec. 3	16	SAT 4 Sec. 3	18
SAT 4 Sec. 4	25	SAT 5 Sec. 3	10	SAT 5 Sec. 4	30
SAT 6 Sec. 3	12	SAT 7 Sec. 3	13	SAT 8 Sec. 3	8
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## 7 Quadratics

### 7.1 Definition

A quadratic is simply a polynomial with degree two. That is, it is when the largest exponent is a 2.

### 7.2 Terminology

Part of the reason many students find quadratics difficult is because of some of the terminology involved. In particular, there are four words that all mean the exact same thing and a fifth word with a related meaning. Students have rarely seen these connections and the designers of the tests know this and design questions based on this misunderstanding. Several questions become incredibly simple once students understand the connection of all these words.

The following four words all mean the exact same thing

- x intercept
- root
- solution
- zero



The word “factor” means one term that is multiplied by other factors to form the polynomial. The factor is closely related to, but distinct, from the words x intercept, root, solution, and zero. When a factor is set equal to zero and solved for x, this reveals the x intercept, root, solution, and zero. Suppose we have the following polynomial

$$y = x(x + 7)(x - 3)(2x - 9)$$

The factors are  $x$ ,  $(x + 7)$ ,  $(x - 3)$ , and  $(2x - 9)$ . When we set each of these equal to zero, we can find each of the x intercepts, roots, solutions, and zeros.

$$x = 0$$

$$x + 7 = 0 \quad x = -7$$

$$x - 3 = 0 \quad x = 3$$

$$2x - 9 = 0 \quad x = \frac{9}{2}$$

Thus the x intercepts, roots, solutions, and zeros are 0,  $-7$ , 3, and  $\frac{9}{2}$ .

Working in reverse, if we are told that the x intercepts, roots, solutions, or zeros of a polynomial are  $-3$ , 2, and 5, we know that the factors are  $(x + 3)$ ,  $(x - 2)$ , and  $(x - 5)$ .

If a factor is squared, it is a solution with multiplicity 2. This means that the polynomial “bounces” off the x-axis at the corresponding root, instead of passing through the x-axis.

### 7.3 Standard Form

The following is the standard form of a quadratic equation.

$$y = ax^2 + bx + c$$

Standard form is the least useful of the three forms. However, there are some useful things we can do with standard form.

- Standard form is the only form that shows us the values of  $a$ ,  $b$ , and  $c$ . These are necessary to use the quadratic formula or determinant (see below).
- Whether the parabola opens upward (a positive  $a$  value) or opens downward (a negative  $a$  value).
- The y intercept (the  $c$  value).

Because standard form is the least useful form, it is important to know how to change from standard form into the other forms.

### 7.4 Intercept Form

The following is the intercept form (also called factored form).

$$y = a(x - m)(x - n)$$

As the name implies, we can determine the x intercepts of a quadratic simply by looking at the intercept form:  $m$  and  $n$ . Specifically, the x intercepts are found by setting each factor equal to zero. Therefore, if we have the following quadratic in intercept form

$$y = (2x + 5)(x - 6)$$

we find the x intercepts by setting each factor equal to zero.

$$2x + 5 = 0$$

$$2x = -5$$

$$x = -\frac{5}{2}$$

and

$$x - 6 = 0$$

$$x = 6$$

Thus the x intercepts are  $-\frac{5}{2}$  and 6.

The  $a$  value is the same as the  $a$  value in standard form and therefore tells us whether the parabola opens upward (a positive  $a$  value) or opens downward (a negative  $a$  value).

To get a quadratic from standard form into intercept form, you simply need to factor the quadratic in standard form.

$$y = x^2 - x - 12$$

$$y = (x - 4)(x + 3)$$

Thus the x intercepts would be 4 and -3.

## 7.5 Vertex Form

The following is the vertex form (also called graphing form).

$$y = a(x - h)^2 + k$$

As the name implies, we can determine the location of the vertex of a parabola simply by looking at the vertex form. Specifically, the vertex is located at  $(h, k)$ . Therefore, if we have the following quadratic in vertex form

$$y = (x - 4)^2 + 1$$

we find that the vertex is located at  $(4, 1)$ .

The  $a$  value is the same as the  $a$  value in standard form and therefore tells us whether the parabola opens upward (a positive  $a$  value) or opens downward (a negative  $a$  value).

In order to get a quadratic from standard form into vertex form, you simply complete the square.

$$y = x^2 - 6x + 7$$

$$y = (x^2 - 6x + \underline{\quad}) + 7 - \underline{\quad}$$

$$y = (x^2 - 6x + \underline{9}) + 7 - \underline{9}$$

$$y = (x^2 - 6x + 9) - 2$$

$$y = (x - 3)^2 - 2$$

Thus the vertex is at  $(3, -2)$ .

## 7.6 Quadratic Formula

The quadratic formula tells us the  $x$  intercepts, roots, solutions, or zeros of a quadratic function in standard form. The  $a$ ,  $b$ , and  $c$  values from standard form are simply plugged into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus, the quadratic formula is an alternative to factoring a quadratic and putting it into intercept form to find the  $x$  intercepts.

This formula is not given on the SAT or ACT but is extremely useful on the test. Therefore it is the most important formula to memorize for the test, if you have not memorized it already.

## 7.7 Discriminant

The discriminant is the portion of the quadratic formula involving the radical

$$\sqrt{b^2 - 4ac}$$

It is a very useful tool when asked *how many*  $x$  intercepts, solutions, roots, or zeros a quadratic has. The only thing that matters is whether the quantity under the radical is positive, zero, or negative – not the specific number.

$\sqrt{+}$  – two solutions

$\sqrt{0}$  – one solution

$\sqrt{-}$  – no real solutions

If we are asked how many solutions the following quadratic in standard form has

$$y = 12x^2 - 7x + 5$$

we simply need to determine whether the discriminant is positive, zero, or negative.

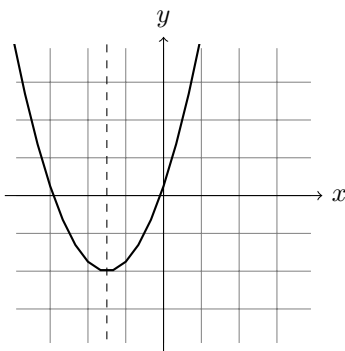
$$\sqrt{(-7)^2 - 4(12)(5)}$$

$$\sqrt{49 - 4(12)(5)}$$

Even without a calculator, this is obviously negative (the actual number does not matter). Therefore there are no real solutions.

## 7.8 Axis of Symmetry

The axis of symmetry is the line across which a parabola is symmetrical (that is, which the parabola could fold across and “land” back on itself).



However, this is not why the axis of symmetry is useful for the SAT and ACT. The reason it is useful is because the axis of symmetry passes through the vertex of the parabola. That is, the axis of symmetry quickly and easily gives us the  $x$  value of the vertex of a parabola.

### 7.8.1 Finding the Axis of Symmetry in Standard Form

When in standard form, the axis of symmetry is given by

$$x = -\frac{b}{2a}$$

Note that this is simply part of the quadratic formula.

If we have the following quadratic in standard form

$$y = 4x^2 - 2x + 3$$

then the axis of symmetry (and thus the  $x$  value of the vertex) is

$$x = \frac{-(-2)}{2(4)} = \frac{2}{8} = \frac{1}{4}$$

### 7.8.2 Finding the Axis of Symmetry in Intercept Form

When in intercept form, the axis of symmetry is found by averaging the two  $x$  intercepts. If we have the following quadratic in intercept form

$$y = (x + 4)(x - 6)$$

then the axis of symmetry (and thus the  $x$  value of the vertex) is

$$x = \frac{-4 + 6}{2} = \frac{2}{2} = 1$$

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 3	16	SAT 1 Sec. 4	25	SAT 1 Sec. 4	29
SAT 1 Sec. 4	30	SAT 2 Sec. 3	13	SAT 2 Sec. 4	7
SAT 2 Sec. 4	29	SAT 3 Sec. 3	7	SAT 3 Sec. 3	12
SAT 3 Sec. 3	14	SAT 3 Sec. 4	12	SAT 4 Sec. 3	11
SAT 4 Sec. 3	13	SAT 4 Sec. 3	15	SAT 4 Sec. 4	12
SAT 4 Sec. 4	28	SAT 5 Sec. 3	3	SAT 5 Sec. 3	4
SAT 6 Sec. 3	4	SAT 6 Sec. 3	11	SAT 6 Sec. 3	14
SAT 6 Sec. 4	20	SAT 6 Sec. 4	30	SAT 6 Sec. 4	34
SAT 7 Sec. 3	7	SAT 7 Sec. 3	10	SAT 7 Sec. 3	12
SAT 7 Sec. 3	15	SAT 7 Sec. 4	6	SAT 7 Sec. 4	24
SAT 7 Sec. 4	35	SAT 8 Sec. 3	11	SAT 8 Sec. 3	14
SAT 8 Sec. 3	16	SAT 8 Sec. 4	8	SAT 8 Sec. 4	14
SAT 8 Sec. 4	19	SAT 8 Sec. 4	35	ACT 2005-06	28
ACT 2005-06	50	ACT 2011-12	21	ACT 2014	59
ACT 2016-17	24	ACT 2016-17	28		

## 8 Complex Numbers

### 8.1 Definition and Strategy

As you probably learned in one of your high school math classes,  $i$  (the imaginary number) is defined to be  $\sqrt{-1}$ . Therefore all complex number problems on the SAT explicitly state

$$i = \sqrt{-1}$$

However, this is a distraction. There are no problems in which you will have to turn  $\sqrt{-1}$  into  $i$  or vice versa. Therefore cross this out. Instead, you should write

$$i^2 = -1$$

This is the only piece of information about complex numbers that you need to know.

There are only three types of complex number problems that you will encounter; there will never be any complicated or tricky questions. All complex numbers can be solved by following these steps.

- If the problem is a division problem, multiply the numerator (top) and the denominator (bottom) by the conjugate of the denominator (bottom). (*See below.*)
- Treat  $i$  like any other variable and perform any operations (such as addition, subtraction, or multiplication) as you normally would.
- Any time an  $i^2$  appears, replace it with  $-1$ .

### 8.2 Addition and Subtraction

Suppose we have the problem

$$(5 - 4i) - (7 - 2i)$$

Following the steps above, we simply treat  $i$  like any other variable and perform the indicated operation (subtraction) by distributing the negative and combining like terms.

$$\begin{aligned}(5 - 4i) - (7 - 2i) \\ 5 - 4i - 7 + 2i \\ -2 - 2i\end{aligned}$$

Because there is no  $i^2$  present, we are done.

### 8.3 Multiplication

Suppose we have the problem

$$(3 - 2i)(4 + i)$$

Following the steps above, we simply treat  $i$  like any other variable and perform the indicated operation (multiplication) by FOILing and combining like terms.

$$\begin{aligned}(3 - 2i)(4 + i) \\ 12 + 3i - 8i - 2i^2 \\ 12 - 5i - 2i^2\end{aligned}$$

Next we simply replace any  $i^2$  with  $-1$ .

$$\begin{aligned}12 - 5i - 2i^2 \\ 12 - 5i - 2(-1) \\ 12 - 5i + 2 \\ 14 - 5i\end{aligned}$$

### 8.4 Division

Suppose we have the problem

$$\frac{2 + 3i}{1 - 4i}$$

Following the steps above, we must multiply the numerator (top) and denominator (bottom) by the conjugate of the denominator (bottom). The conjugate of a complex number is simply the same complex number, except with the sign in front of the  $i$  changed. Here, the conjugate of the denominator (bottom) is therefore  $1 + 4i$ .

Therefore we multiply as follows:

$$\frac{2 + 3i}{1 - 4i} \cdot \frac{1 + 4i}{1 + 4i}$$

Now we simply follow the rest of the steps above for complex number problems. We are now multiplying two fractions, so we treat  $i$  like any other variable and multiply the two numerators (tops) and the two denominators (bottoms) by FOILing and combining like terms.

$$\frac{2 + 3i}{1 - 4i} \cdot \frac{1 + 4i}{1 + 4i}$$

$$\frac{2 + 8i + 3i + 12i^2}{1 + 4i - 4i - 16i^2}$$

$$\frac{2 + 11i + 12i^2}{1 - 16i^2}$$

Finally, we simply replace any  $i^2$  with  $-1$ .

$$\frac{2 + 11i + 12i^2}{1 - 16i^2}$$

$$\frac{2 + 11i + 12(-1)}{1 - 16(-1)}$$

$$\frac{2 + 11i - 12}{1 + 16}$$

$$\frac{-10 + 11i}{17}$$

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 3	2	SAT 1 Sec. 3	11	SAT 4 Sec. 3	14
SAT 6 Sec. 3	3	SAT 7 Sec. 3	4	ACT 2005-06	46
ACT 2016-17	29				

## 9 Geometry

### 9.1 Introduction

There are no pure geometry problems on the SAT or ACT. Instead, all geometry problems can and should be turned into algebra problems easily and quickly. Only a few basic geometry concepts are needed to do this. If you find yourself delving into advanced geometry concepts or doing a proof, you are not approaching the problem correctly. Instead, turn it into an algebra problem.

Generally this can be done by labeling the unknown quantity that you want to know with a variable (a new variable if the problem already has other variables), using a geometry property to turn the problem into an algebra equation, and then solving for the variable you want to know.

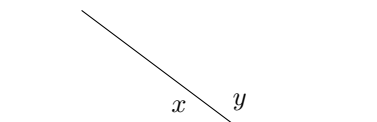
### 9.2 Triangle Angles

All the angles in a triangle add to be 180 degrees. This is a useful property for turn a problem into an algebra equation.

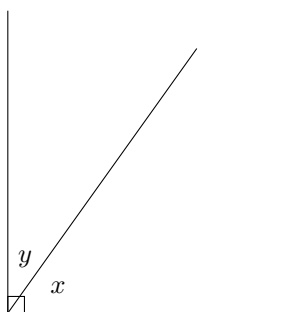
The largest angle is always across from the longest side; the smallest angle is always across from the shortest side.

### 9.3 Complimentary and Supplementary Angles

If two angles are complimentary, they add to be 90 degrees. If two angles are supplementary, they add to be 180 degrees. In either situation, an algebraic equation should be set up immediately.



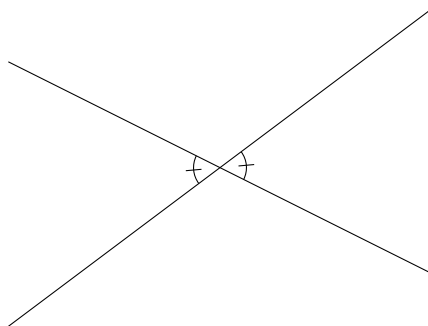
Here, the angles are supplementary. Therefore angles should immediately be labeled and the algebraic equation  $x + y = 180$  can be used.



Here, the angles are complimentary. Therefore, angles should immediately be labeled and the algebraic equation  $x + y = 90$  can be used.

### 9.4 Vertical Angles

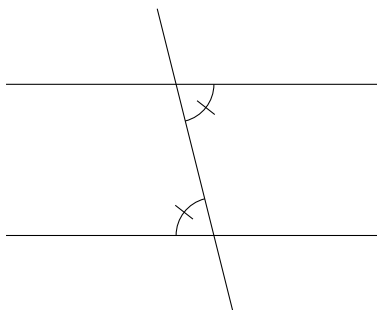
When two (or more) lines intersect, the angles across the intersection point from each other are equal (congruent). Any vertical angles should immediately be marked as equal.





## 9.5 Parallel Lines

Parallel lines are not of any use in and of themselves on the SAT or ACT. The only reason parallel lines are useful is because alternate interior angles are congruent. Therefore, whenever you are told that lines are parallel (whether you are told directly, it is indicated in a diagram, or implied by a particular shape such as a trapezoid or parallelogram), you should immediately think of alternate interior angles.



Alternate interior angles are the angles that occur on opposite (*alternate*) sides of the transversal (the line that cuts through the two parallel lines) but inside (*interior*) two parallel lines.

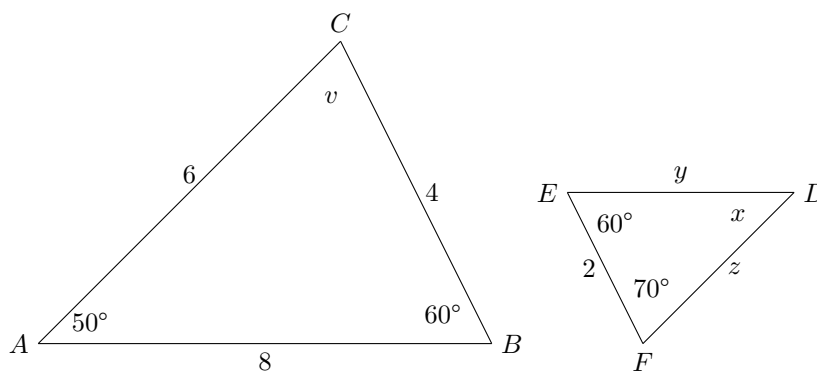
Using alternate interior angles and vertical angles, several angles can be related to each other.

Sometimes alternate interior angles are hidden. However, they are still there. To uncover them, the parallel lines and the transversal should be extended.

## 9.6 Similar Triangles

If two triangles are similar, it means two things.

- All the corresponding angles are congruent.
- All the corresponding sides are proportional.



First we solve for  $v$  and  $x$ :

$$50 + 60 + v = 180$$

$$v = 70$$

$$60 + 70 + x = 180$$

$$x = 50$$

This means the triangles are similar because all the angles are the same. Specifically,  $\triangle ABC \sim \triangle DEF$ .

Therefore, the sides are proportional and we can solve for  $y$  and  $z$ .

$$\frac{4}{2} = \frac{8}{y}$$

$$4y = 16$$

$$y = 4$$

$$\frac{4}{2} = \frac{6}{z}$$

$$4z = 12$$

$$z = 3$$

## 9.7 Special Types of Triangles

### Equilateral Triangles

Equilateral triangles have the following properties:

- All three side lengths are the same.
- All three angle measures are the same ( $60^\circ$ ).

The formula for the area of an equilateral triangle is useful to know.

$$A = \frac{\sqrt{3}}{4}s^2$$

where  $s$  is the length of a side of the equilateral triangle. If you don't remember the formula for the area of an equilateral triangle, it can be derived easily using the 30-60-90 special triangle (see below).

### Isosceles Triangles

Isosceles triangles have the following properties:

- Two side lengths are the same.
- Two angle measures are the same.

It is important to remember that if you know one of these items is true, you know that the other is true as well.

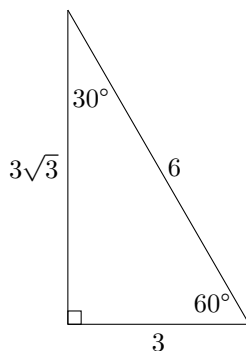
### 30-60-90 Triangles

A 30-60-90 triangle is a right triangle in which the other two angles are  $30^\circ$  and  $60^\circ$ . 30-60-90 triangles are particularly useful because they result from splitting an equilateral triangle in half. The formula for 30-60-90 triangles is given to you on the SAT.

To use the 30-60-90 formula, side lengths should only be compared to the side across from  $30^\circ$ .

To get the hypotenuse, multiply the side across from  $30^\circ$  by 2. To get the side across from  $60^\circ$ , multiply the side across from  $30^\circ$  by  $\sqrt{3}$ .

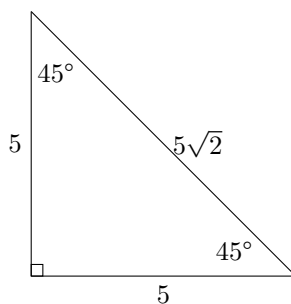
If you have the hypotenuse, divide it by 2 to get the side across from  $30^\circ$ . If you have the side across from  $60^\circ$ , divide it by  $\sqrt{3}$  to get the side across from  $30^\circ$ .



### 45-45-90 Triangles

A 45-45-90 triangle is a right triangle in which the other two angles are equal. Thus, it is an isosceles triangle. Therefore, the other two angles are each  $45^\circ$ . The formula for 45-45-90 triangles is given to you on the SAT.

Each of the two legs will be equal in length. If you have the length of a leg, simply multiply it by  $\sqrt{2}$  to get the length of the hypotenuse. If you have the length of the hypotenuse, simply divide it by  $\sqrt{2}$  to get the length of the legs.



### Pythagorean Theorem and Pythagorean Triples

You must know the Pythagorean theorem.

$$a^2 + b^2 = c^2$$

where  $a$  and  $b$  are the legs of a right triangle and  $c$  is the hypotenuse of the right triangle.

Pythagorean triples are three whole numbers that work together in the Pythagorean theorem. The most common Pythagorean triple is 3 – 4 – 5. The second most common Pythagorean triple is 5 – 12 – 13. Any multiple of a Pythagorean triples also works. Therefore, 9 – 12 – 15 is also a Pythagorean triple. Recognizing Pythagorean triples on the test can save a lot of time.

## 9.8 Degrees in a Polygon

The number of degrees in a polygon with  $n$  sides is given by

$$(n - 2)180$$

If the polygon is *regular* this means

- All side lengths are the same.
- All angle measures are the same.

Therefore, in a regular polygon, each interior angle is

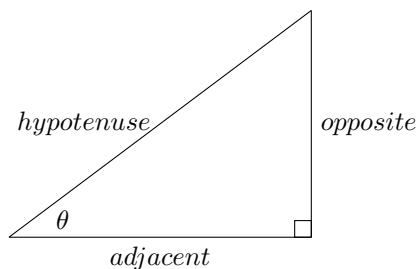
$$\frac{(n - 2)180}{n}$$

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 3	17	SAT 1 Sec. 4	3	SAT 2 Sec. 3	18
SAT 2 Sec. 3	19	SAT 2 Sec. 4	30	SAT 3 Sec. 3	11
SAT 3 Sec. 3	18	SAT 4 Sec. 3	16	SAT 5 Sec. 3	20
SAT 5 Sec. 4	19	SAT 5 Sec. 4	26	SAT 6 Sec. 3	18
SAT 6 Sec. 4	16	SAT 7 Sec. 3	17	SAT 7 Sec. 4	14
SAT 8 Sec. 3	4	SAT 8 Sec. 4	5	ACT 2005-06	6
ACT 2005-06	12	ACT 2005-06	13	ACT 2005-06	20
ACT 2005-06	23	ACT 2005-06	24	ACT 2005-06	25
ACT 2005-06	33	ACT 2005-06	36	ACT 2005-06	37
ACT 2005-06	58	ACT 2008-09	9	ACT 2008-09	14
ACT 2008-09	15	ACT 2008-09	16	ACT 2008-09	18
ACT 2008-09	35	ACT 2008-09	39	ACT 2008-09	44
ACT 2008-09	45	ACT 2008-09	46	ACT 2008-09	58
ACT 2011-12	5	ACT 2011-12	10	ACT 2011-12	12
ACT 2011-12	16	ACT 2011-12	20	ACT 2011-12	25
ACT 2011-12	34	ACT 2011-12	39	ACT 2011-12	43
ACT 2011-12	44	ACT 2011-12	59	ACT 2014-15	11
ACT 2014-15	17	ACT 2014-15	27	ACT 2014-15	30
ACT 2014-15	31	ACT 2014-15	38	ACT 2014-15	40
ACT 2014-15	47	ACT 2016-17	12	ACT 2016-17	13
ACT 2016-17	17	ACT 2016-17	20	ACT 2016-17	26
ACT 2016-17	27	ACT 2016-17	40	ACT 2016-17	41
ACT 2016-17	44	ACT 2016-17	50	ACT 2005-06	54
SAT 8 Sec. 4	10	ACT 2008-09	27	ACT 2014-15	23
ACT 2014-15	44				

## 10 Trigonometry

### 10.1 Trigonometric Ratios

You probably remember the term “SOH-CAH-TOA” for remembering trigonometric ratios in a right triangle. It is important to be able to set up these ratios.



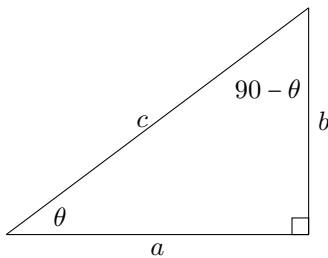
$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

## 10.2 Complimentary Angle Theorem

If two angles are complimentary, then the sine of the first angle and the cosine of the second angle are equal. For example:

$$\sin(37) = \cos(53)$$

This is easily shown in a right triangle.



If we compare sine of  $\theta$  and cosine of  $\theta - 90$ , we will see that they are equal.

$$\sin(\theta) = \frac{b}{c} \quad \cos(90 - \theta) = \frac{b}{c}$$

## 10.3 Law of Sines and Law of Cosines

On the ACT, the Law of Sines and Law of Cosines are occasionally tested. These laws are used to solve non-right triangles. The formulas will be given to you and you only have to set them up – not solve them. Therefore, you only need to know which law to choose.

- If you know a side across from an angle, use Law of Sines.
- If not, use Law of Cosines.

Test	Number	Test	Number	Test	Number
SAT 1 Sec. 3	19	SAT 3 Sec. 3	20	SAT 3 Sec. 4	23
SAT 4 Sec. 3	17	SAT 7 Sec. 3	18	SAT 7 Sec. 4	36
SAT 8 Sec. 4	36	ACT 2005-06	34	ACT 2005-06	45
ACT 2005-06	50	ACT 2005-06	52	ACT 2008-09	25
ACT 2008-09	32	ACT 2008-09	55	ACT 2008-09	56
ACT 2011-12	38	ACT 2011-12	41	ACT 2011-12	53
ACT 2014-15	22	ACT 2014-15	42	ACT 2014-15	56
ACT 2014-15	57	ACT 2016-17	30	ACT 2016-17	57
ACT 2016-17	60	SAT 5 Sec. 4	20		

## 11 Circles

### 11.1 Basic Formulas

On the ACT, you need to know the formulas for the circumferences (perimeter) of a circle and for the area of a circle. These formulas are given to you on the SAT.

$$C = 2\pi r$$

$$A = \pi r^2$$

If you are given a point on a circle, the best way to approach the problem is to draw a line from the point to the x axis to create a triangle.

You will also need to know the general graphing form of a circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where  $(h, k)$  is the center of the circle and  $r$  (not  $r^2$ ) is the radius of the circle. Thus the following circle would have its center at  $(-5, 1)$  and would have a radius of 9.

$$(x + 5)^2 + (y - 1)^2 = 81$$

If you are given an equation with both an  $x^2$  term and a  $y^2$  term, this is a circle. To put it in graphing form, you need to complete the square twice: once for  $x$  and once for  $y$ . Completing the square twice below would result in the same equation of the circle above in graphing form.

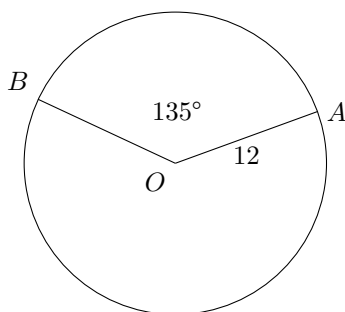
$$x^2 + y^2 + 10x - 2y = 55$$

### 11.2 Arc Length

Arc length is simply a portion of the circumference (perimeter) of a circle. It is determined by setting up a ratio between the arc length and the total circumference (perimeter) on the one hand, and the central angle and the total number of degrees ( $360^\circ$ ) on the other hand.

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{360^\circ}$$

If we want to find the length of arc  $\widehat{AB}$ :



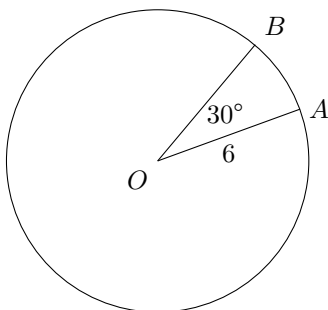
$$\begin{aligned}\frac{x}{2\pi 12} &= \frac{135^\circ}{360^\circ} \\ 360x &= 3240\pi \\ x &= 9\pi\end{aligned}$$

### 11.3 Sector Area

Sector area is simply a portion of the area of a circle. To determine the sector area, we simply set up a ratio between the sector area and the total area of the circle on the one hand, and the central angle and the total number of degrees ( $360^\circ$ ) on the other hand.

$$\frac{\text{sector area}}{\text{area}} = \frac{\text{central angle}}{360^\circ}$$

If we want to find the area of  $\triangle AOB$ :

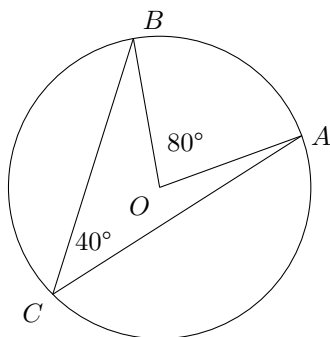


$$\begin{aligned}\frac{x}{\pi 6^2} &= \frac{30^\circ}{360^\circ} \\ 360x &= 1080\pi \\ x &= 3\pi\end{aligned}$$

### 11.4 Central Angles and Inscribed Angles

A central angle is an angle with its vertex at the center of a circle. An inscribed angle is an angle with its vertex on the edge of the circle. If a central angle and an inscribed angle point to the same arc, then the central angle will be twice the inscribed angle.





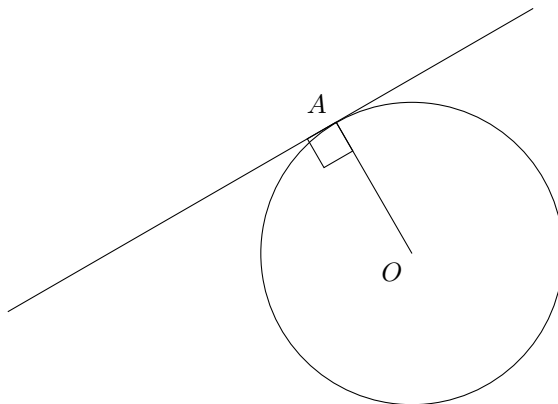
Here,  $\angle BOA$  is a central angle and  $\angle BCA$  is an inscribed angle. Therefore,  $\angle BOA$  is twice  $\angle BCA$ .

### 11.5 Central Angle Measure

If you are asked to find the central angle measure, you simply need to use the ratios described above for arc length or sector area (depending on what you are given) and then solve for the central angle measure.

### 11.6 Tangent Lines

A line that is tangent to a circle is a line that touches the circle at exactly one point. In addition, The tangent line will be perpendicular to the radius from the center of the circle to the point of tangency. Therefore, the angle should immediately be marked as being  $90^\circ$ .



Test	Number	Test	Number	Test	Number
SAT 1 Sec. 4	24	SAT 2 Sec. 4	24	SAT 2 Sec. 4	36
SAT 3 Sec. 4	34	SAT 4 Sec. 4	24	SAT 4 Sec. 4	36
SAT 5 Sec. 3	2	SAT 5 Sec. 4	29	SAT 5 Sec. 4	36
SAT 6 Sec. 3	20	SAT 6 Sec. 4	27	SAT 7 Sec. 4	29
SAT 7 Sec. 4	34	SAT 8 Sec. 3	9	SAT 8 Sec. 3	20
ACT 2005-06	26	ACT 2005-06	31	ACT 2005-06	38
ACT 2005-06	60	ACT 2008-09	30	ACT 2008-09	54
ACT 2011-12	15	ACT 2011-12	37	ACT 2011-12	54
ACT 2011-12	55	ACT 2014-15	36	ACT 2014-15	37
ACT 2014-15	45	ACT 2014-15	48	ACT 2016-17	14
ACT 2014-15	18				

## 12 Statistics and Probability

Many students have not had any exposure to statistics and feel overwhelmed by the statistics questions on the SAT and the ACT. While there are statistical concepts on the SAT and ACT, they are all cursory and easy to master.

### 12.1 Counting Problems

If asked to determine how many possibilities there are in a scenario, simply draw a line for each decision that must be made. Then write the number of options there are for each slot/decision. Finally, multiply all of these together.

If someone must choose an outfit from 4 pairs of shoes, 5 pairs of pants, and 8 shirts, how many possibilities are there?

— — —

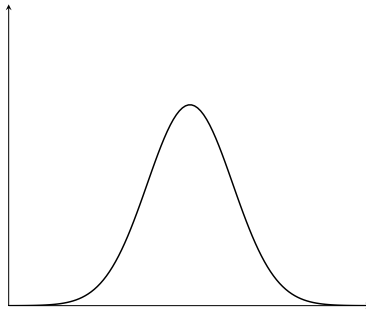
$$\underline{4 \cdot 5 \cdot 8}$$

160

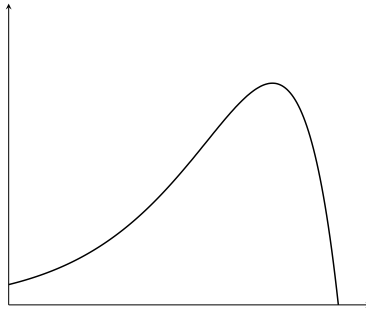
### 12.2 Distributions

A distribution is simply a visual representation of how data is spread out, such as with a histogram (bar chart).

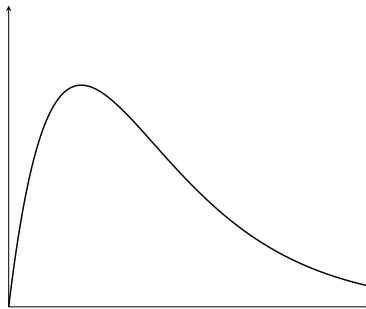
**Normal distribution** A normal distribution is one that looks like this. It is symmetrical across its center. The ends that trail off on either side are known as “tails.”



**Skew** Skew occurs when one of the tails of a distribution is pulled out or elongated. If the right tail is pulled out or elongated, it is “right skew.” If the left tail is pulled out or elongated, it is “left skew.” Skew is about an overall trend in the data, not individual data points.

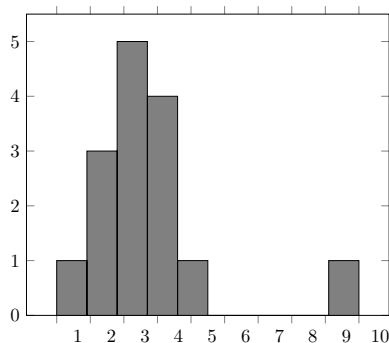


*Left skew*



*Right skew*

**Outliers** Outliers are similar to skew in that they represent the data stretching out in one direction. But it is about individual data points lying outside the general trend of the data. In the following graph, 9 is an outlier.



### 12.3 Measures of Center

Measures of center are various ways to determining what the center or middle of a group of data is.

**Mean** The mean of data is just the average of all the data points. That is, it can be calculated by adding all the data points together and dividing by the number of data points. The key thing to remember about the mean is that it is *not* resistant to outliers or skew. That is, the mean will move towards any outliers or towards the skewed tail of a distribution. Therefore if there is skew or if there are outliers, the mean is a bad measure of center to use.

**Median** The median is the middle data point *when the data points have been arranged from least to greatest*. Students frequently forget to arrange the data points from least to greatest first. If there is an even number of data points, the median is the average of the middle two data points. The key thing to know about the median is that it *is* resistant to outliers and skew. Therefore, if there is skew or if there are outliers, it is a better measure of center than the mean.

**Mode** The mode is simply the most frequently occurring data point. If there are multiple data points that are repeated the same number of times, then each of them is the mode. The mode is not a particularly useful measure of center and should only be used if you are specifically asked for it.

**Expected Value** Expected value is computed by multiplying the value of each outcome by its probability and then adding all of these numbers.

Prize	Probability
\$0	0.3
\$1	0.4
\$5	0.2
\$10	0.1

Here the expected value would be  $\$0(0.3)+\$1(.4)+\$5(.2)+\$10(.1)=\$0.00+\$0.40+\$1.00+\$1.00=\$2.40$

**Getting the Mean and Median from a Table or Graph** If you are presented with a table or chart with data, finding the mean and the median is easy if you know what you're looking for.

- To find the median, you just need to find the middle item in the already organized data.
- To find the mean, you need to determine what would be in the numerator (top) and denominator (bottom) of your fraction and then use the table to get this data.

Number of Pets	Students
0	12
1	10
2	7
3	3
4	2
5	1
Total	35

There are 35 students, therefore the 18th student will be the middle student. There are 12 students with 0 pets and then there are 10 students with 1 pet, which means we have passed over 22 students. Therefore, the 18th student will be in the group with 1 pet and the median number of pets will be 1.

To determine the mean, we need to figure out what we're looking for. If we're looking for the mean number of pets per student, the fraction will look like this

$$\frac{\text{total number of pets}}{\text{number students}}$$

We know that there are 35 students. To determine the total number of pets, we simply multiply each number of pets by the number of students in that category.

$$\frac{0(12) + 1(10) + 2(7) + 3(3) + 4(2) + 5(1)}{35} = \frac{46}{35} = 1.314$$

## 12.4 Measures of Spread

Measures of spread are various ways of determining how spread out or how clumped together data is.

**Range** Range is simply the largest data point (the max) minus the smallest data point (the min). Thus it is easy to calculate. However, it is not particularly useful since it only compares two of the data points.

**Inter Quartile Range (IQR)** The inter quartile range (IQR) is subtracting the first quartile ( $Q_1$ ) from the third quartile ( $Q_3$ )

$$Q_3 - Q_1$$

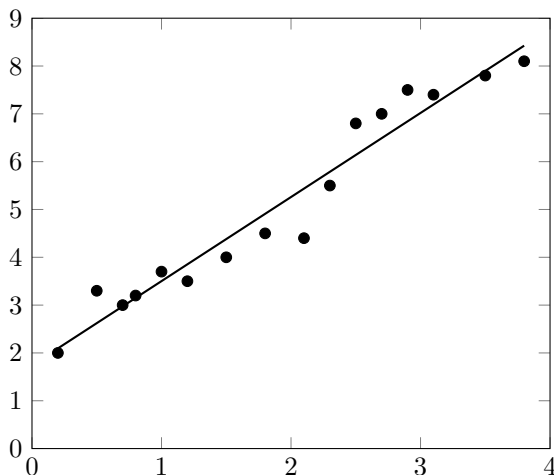
The first quartile ( $Q_1$ ) is the median of the minimum and the median. That is, it is the middle value between the minimum and the median.

The third quartile ( $Q_3$ ) is the median of the median and the maximum. That is, it is the middle value between the median and the maximum.

**Standard Deviation or Variance** Standard deviation and variance are more technical measures of the spread of data. You will not have to calculate standard deviation or variance on the SAT or the ACT. However, you can think of the standard deviation and variance as being the average distance of the data points from the center of the data. Thus a large standard deviation or variance means the data is more spread out; a small standard deviation or variance means the data is more closely clumped around the center of the data.

## 12.5 Regression and Scatterplots

Scatterplots are plots of data that has two variables, such as *hours spent studying for the SAT* and *score on the SAT*.



Each point in the scatterplot is an actual data point that represents an observation.

The line passing through the scatterplot is known as the line of best fit. It describes the data and can be used to predict what one variable will be if the other variable is known.

The residual is the distance between an actual data point and what the line of best fit predicts for the  $y$  variable.

There are several terms that can be used to describe a scatterplot.

- Data is *linear* if it generally fits into a straight line. Data is *nonlinear* if it is generally curved.
- A scatterplot has *strong association or correlation* if it falls close to the line of best fit. It has *weak association or correlation* if it is more spread out from the line of best fit.
- A scatterplot has a *positive association* if the slope of the line of best fit is positive. It has a *negative association* if the slope of the line of best fit is negative.

## 12.6 Basic Statistical Concepts

There are a few statistical concepts that you need to have a basic understanding of.

**Sampling Bias** Sampling bias occurs when the people you choose to question are not *truly* random. That is, if the people you choose to question are not *truly* random, then there is bias.

For example, if you want to gather statistics about a city and stand on a street corner and ask questions of everyone who walks by, this may appear random at first. But it is not actually random because only certain segments of the city population may pass by that street corner.

**Survey Bias** Survey bias occurs when the way that you ask questions in your survey influences how people respond to the questions.

**Inference** Inference is the concept that the statistics you gather cannot apply beyond the group that you drew your sample from. For example, if you do a random survey of people at your school, you cannot use that data to draw inferences about everyone in your city because the two populations (people at your school and people in your city) are different.

## 12.7 Probability

Probability of an event  $A$  is defined as follows:

$$P(A) = \frac{\text{number of ways to get what you want}}{\text{total number of possibilities}}$$

**“And” Probability** The probability of two independent events both occurring is as follows:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

The probability of flipping a Heads on a coin *and* rolling a 6 on a die is as follows:

$$P(H \text{ and } 6) = P(H) \cdot P(6) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$

**“Or” Probability** The probability of either one event or another event occurring is as follows:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

The probability of flipping a Heads on a coin *or* rolling a 6 on a die is as follows:

$$P(H \text{ or } 6) = P(H) + P(6) - P(H \text{ and } 6) = \frac{1}{2} + \frac{1}{6} - \frac{1}{12} = \frac{7}{12}$$

## 12.8 Table Data

When data is presented in a table and the question asks the probability of choosing a certain item from the table, careful attention must be given to the numerator (top) and the denominator (bottom) of the fraction.

	Has pet	No pet	Total
Boy	24	16	40
Girl	12	18	30
Total	36	34	70

These questions must be read carefully because subtle wording can change the meaning of the question significantly. The trick is to focus carefully on what the denominator (bottom) and numerator (top) will be.

What is the probability that someone chosen at random has a pet?  $\frac{36}{70}$

What is the probability that someone chosen at random is a girl with no pet?  $\frac{18}{30}$

What is the probability that a boy chosen has a pet?  $\frac{24}{40}$

Given that someone chosen has a pet, what is the probability that they are a girl?  $\frac{12}{36}$



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ACT 2014-15	12	ACT 2014-15	32	ACT 2014-15	43
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ACT 2016-17	31	ACT 2016-17	37	ACT 2016-17	47
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